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BLADED MACHINES AND EJECTORS (CHAPTERS 1-5)

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BLADED MACHINES AND EJECTORS
(Chapters 1-5)

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13. ABSTRACT

Development of a theory of two-phase gas-liquid and liquid-gas ejectors with a cylindrical mixing chamber for the case where the vapor concentration in the mixture forming at the ejector outlet is negligibly small. The ejection equations, which give the relation backcurrent region can form near the flow core in the exit cone of a supersonic nozzle both in the case of a rotating gas flow and in the case of free swirling flow. The formation of such a region leads to a complex shock wave interaction pattern in the nozzle, which affects flow separation from the wall and the thrust characteristics of the nozzle.
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* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

THEORY OF A TWO-PHASE GAS-LIQUID EJECTOR WITH A CYLINDRICAL MIXING CHAMBER

Yu. N. Vasil'yev

INTRODUCTION

A gas-liquid or liquid-gas ejector is a two-phase ejector in which a liquid enters one nozzle and gas enters another. In the general case the process of mixing the working bodies in a gas-liquid ejector can be accompanied by phase and chemical transformations. At the ejector outlet, depending on the physical properties and the parameters of the state of the gas and the liquid, one of the following mixtures can be formed: one-phase vapor-gas, two-phase vapor-gas-liquid, three-phase gas-vapor-liquid with solid particles, or, finally, a two-phase gas-vapor mixture with solid particles. In a case where the gas is dissolved in the liquid a single-phase solution of gas in liquid may develop at the ejector outlet.

At the present time there is no general theory of a gas-liquid ejector. The most studied case is that of a vacuum liquid-gas ejector with a two-phase gas-liquid mixture. This ejector has been used in various fields of technology for fifty years. A great number of works have been dedicated to it, although because of the

complexity of the mixture-formation process and the uncertainty concerning the properties of the gas-liquid mixture obtained, it has not yet been possible to create even a qualitative theory for this ejector. Existing methods of calculating a liquid vacuum ejector (see [2-4, 12, 13, 15, 16, 19-21]) are empirical and can only be used in a relatively narrow range of change in the parameters which describe its operation. These methods are not sufficiently reliable and in a number of cases provide rather deviant results.

Unsuccessful attempts to create a theory for a vacuum liquid-gas ejector based on equations of the conservation of mass, energy, and momentum are explained by a number of authors by the fact that the mass of ejected gas in such an ejector is thousands of times less than the mass of the ejecting liquid and thus cannot noticeably influence its velocity. This idea has been so convincing that it has been passed on unchanged from article to article (see, for example, works [3, 12, 16]). For this reason the theory of the liquid-gas ejector has remained virtually undeveloped in recent years and all efforts have been directed toward the study of the physics of the phenomenon (see [5, 11]) and the creating of constantly improved empirical calculation methods.

Dedicated to the theoretical study of a gas-liquid ejector with a two-phase gas-liquid mixture is a short article [17] in which ejection equations are derived with the assumption that there is no heat exchange between the liquid and the gas and in which the cut-off regime of the combustion chamber is examined. An attempt was also made to develop a method of calculating a gas-liquid ejector with a two-phase gas-liquid mixture in [16]. Calculation equations for a gas-liquid ejector binding the parameters of the flows in the inlet and outlet sections of the mixing chamber were obtained by the authors using a formal transformation of gas ejector equations under the condition of an incompressible ejector medium. However, this simplified method does not give full consideration to the distinguishing characteristics of the flow of a two-phase

mixture formed at the outlet from the mixing chamber, and thus the obtained equations can only be used in certain particular cases where the parameters of the mixed media are assigned. In the general case the use of the equations of [16] might lead to erroneous results. In particular this applies to the calculation for limiting regimes of ejector operation, which in most cases are the most advantageous.

The present article presents a theory on two-phase gas-liquid and liquid-gas ejectors with cylindrical mixing chambers for a case where a two-phase gas-liquid mixture is formed at the ejector outlet. Ejection equations are derived which show the connection between the flow parameters of the gas in the liquid on the nozzle edge and the parameters of the two-phase mixture in the outlet section of the mixing chamber. Possible operational modes for an ejector are described and classified. Possible systems are examined for the flow of gas and liquid in the nozzles and in the initial part of the mixing chamber and the conditions which connect the flow parameters in different regimes are found.

The ejection equations are derived under the assumption that the two-phase gas-liquid mixture forming in the ejector is homogeneous and that the particles of liquid and the gas are in thermal and mechanical equilibrium. The article does not examine the problem of the cases in which practical satisfaction of these conditions would be possible.

It should be mentioned that the proposed theory does not enable calculating ejectors in which, for one reason or another, (for example, because of insufficient length of the mixing chamber) the gas-liquid mixture does not satisfy the above mentioned conditions. However, the efficiency of such ejectors is always lower than for the case of a homogeneous and equilibrium mixture. Thus, using the equation which has been obtained we can estimate the limiting characteristics of the two-phase ejector. Comparison of experimental characteristics with limiting theoretical

characteristics lets us judge the degree of perfection of the studied ejector and the possibilities of improving its effectiveness. Furthermore, a qualitatively correct description of the processes which occur in the flow part of the ejector enable us to take direct measures in improving its parameters.

Testing the theory using a vacuum liquid-gas ejector as the model gave encouraging results (see article [8] of this collection). Methods based upon it enabled a substantial increase in the efficiency of an ejector. In this case the experimental characteristics of improved variants of the ejector, which provided more complete mixing of the flows and breaking of the liquid jet, were in satisfactory agreement with theoretical characteristics over a wide range of variation in the parameters of the gas and the liquid.

Calculations made from theoretical dependences of the present work indicated that just as in gas ejectors with high-pressure drops, the most advantageous operational regime for vacuum liquid-gas ejectors are critical regimes in which the flow of the two-phase mixture in the outlet section of the mixing chamber is supersonic. Losses in the direct plane shock, which transforms the supersonic flow of the two-phase mixture into a subsonic flow, comprise most of the losses which determine the efficiency of such an ejector in limiting critical regimes with the expanding diffuser which is usually used. Thus, it was proposed that the expanding diffuser should be replaced by a supersonic diffuser with a throat, in which damping of the supersonic flow in the two-phase mixture would occur in systems of angle shocks, cut off by the plane shock, with much lower losses. This idea was shown to be correct by an experimental test on a series of vacuum ejectors.

In this case the total pressure of the mixture at the ejector outlet and its efficiency increased 1.5-2.0 times (see [8]). Thus, even the first attempts to use the results of

this work enabled a considerable increase in the efficiency of liquid-gas ejectors and pointed the way to further improvements.

CHAPTER 1

GENERAL PROPERTIES OF EQUILIBRIUM TWO-PHASE GAS-LIQUID MIXTURE. CALCULATING ONE-DIMENSIONAL FLOWS

In the present work, as we have already mentioned, we are studying a gas-liquid ejector in which a two-phase gas-liquid mixture is formed at the outlet from the mixing chamber. In deriving ejection equations and calculating the flow of the obtained two-phase mixture in the diffuser the following assumptions were made:

1) the drops of liquid are equally distributed throughout the entire mixture, and their dimensions are so small that in the process of changing state they are in thermal and mechanical equilibrium with the gas;

2) changes in state of the mixture are not accompanied by chemical transformations;

3) there is no mass exchange between the phases (evidently the mixture satisfies this condition if the gas contained in it does not dissolve in the liquid and if in the studied range of temperature variation the saturation pressure of the liquid is much less than the pressure of the gas);

4) the gas is ideal and its physical constants do not depend on temperature or pressure;

5) the heat capacity and specific weight of the liquid do not depend on temperature or pressure.

Now we introduce the designations: p - pressure per kgf/m^2; T in °K and t in °C - temperature; γ in kgf/m^3 - specific weight; ρ in $kg \cdot s^2/m^4$ - density; v in m^3/kg - specific volume; i in Cal/kg - enthalpy; U in Cal/kg - internal energy; s in Cal/(kg × × deg) - entropy; c in Cal/(kg·deg) - specific heat capacity (c_p and c_v - specific heat capacities under constant pressure and volume, respectively); R in kgf·m/(kg·deg) - gas constant; J in kgf·m/Cal - mechanical equivalent of heat; g in m/s^2 - gravitational acceleration; G in kgf/s - per-second mass flow rate; f in m^2 - area of cross section of stream; W in m/s - velocity of flow; $a_{к}$ in m/s - critical velocity; a in m/s - speed of sound; $\lambda = W/a_{к}$ - reduced velocities; $M = W/a$ - M number; $\kappa = c_p/c_v$ - adiabatic exponent; $K = G_{г}/G_{ж}$ - ratio of mass flow rates of gas and liquid in mixture, equal to the ejection coefficient. Quantities with the subscript "c" correspond to the parameters of the gas-liquid mixture; quantities with subscripts "г," "п," and "ж," correspond to the parameters of the gas, vapor, and liquid. Subscripts with "0" and "s" denote the parameters of stagnation and saturation; subscript "к" - the parameters of the flow in the critical section of the channel, where $W = a = a_{к}$.

1.1. Physical Constants and Parameters of State of the Mixture. Equation of State

The properties of an equilibrium two-phase gas-liquid mixture under the assumptions made above has been studied in a number of works (see, for example, [14]). Thus, in this article the problem is not thoroughly examined; below we present only those relationships which are necessary for the following analysis of the work of the ejector.

The specific heat capacities and the adiabatic exponents of the studied two-phase mixture, as can be easily demonstrated, are determined as follows:

$$c_{p\,\text{с}}=\frac{Kc_{p\,\text{г}}+c_{\text{ж}}}{K+1}; \tag{1.1}$$

$$c_{v\,\text{с}}=\frac{Kc_{v\,\text{г}}+c_{\text{ж}}}{K+1}; \tag{1.2}$$

$$\gamma_{\text{с}}=\frac{c_{p\,\text{с}}}{c_{v\,\text{с}}}=\frac{Kc_{p\,\text{г}}+c_{\text{ж}}}{Kc_{v\,\text{г}}+c_{\text{ж}}}. \tag{1.3}$$

The specific weight of the mixture is found from the obvious relationship:

$$\frac{G_{\text{ж}}}{\gamma_{\text{ж}}}+\frac{G_{\text{г}}}{\gamma_{\text{г}}}=\frac{G_{\text{с}}}{\gamma_{\text{с}}}, \tag{1.4}$$

which, considering equality

$$G_{\text{с}}=G_{\text{г}}+G_{\text{ж}}=G_{\text{ж}}(K+1), \tag{1.5}$$

leads to form

$$\gamma_{\text{с}}=\frac{K+1}{\frac{K}{\gamma_{\text{г}}}+\frac{1}{\gamma_{\text{ж}}}}. \tag{1.6}$$

If we consider that the specific weight of the liquid does not depend on temperature and pressure, while the specific weight of the gas is determined by the Clapeyron equation

$$p_{\text{с}}=R_{\text{г}}\gamma_{\text{г}}T_{\text{с}}, \tag{1.7}$$

then from equation (1.6) we obtain the equation of state of a two-phase gas-liquid mixture:

$$p_{\text{с}}\left[1-\frac{\gamma_{\text{с}}}{\gamma_{\text{ж}}(K+1)}\right]=\frac{K}{K+1}R_{\text{г}}\gamma_{\text{с}}T_{\text{с}}. \tag{1.8}$$

If we introduce

$$R_c = \frac{K}{K+1} R_г \quad (1.9)$$

then we write equation (1.8) in the form of

$$p_c \left[1 - \frac{\gamma_c}{\gamma_ж (K+1)}\right] = R_c \gamma_c T_c. \quad (1.10)$$

Equation (1.10) is distinguished from the equation of state of an ideal gas with gas constant R_c only by the presence of the term $\gamma_c/[\gamma_ж(K + 1)]$, which is equal to the relative volume of the liquid in the mixture $V_ж/V_c$. In the case of gas concentrations which are not very low and relatively low pressures the volume of liquid is not great, and thus the behavior of the studied two-phase mixture is very close to that of an ideal gas with gas constant R_c. For example, for a water-air mixture at $K > 0.05$, $p_c < 2$ kgf/cm^2, and $T_c = 288°K$ error in determining p_c and γ_c without considering the volume of the liquid contained in the mixture does not exceed 4.5%.

From equation (1.9) using equations (1.1), (1.2), and (1.3), and considering that an ideal gas is subject to the Meyer equation

$$R_г = J(c_{pг} - c_{vг}), \quad (1.11)$$

we find

$$R_c = J(c_{pc} - c_{vc}) = Jc_{vc}(\varkappa_c - 1) = Jc_{pc}\frac{\varkappa_c - 1}{\varkappa_c}. \quad (1.12)$$

Thus, it follows that the studied two-phase mixture is also subject to the Meyer equation, and, consequently, its internal energy does not depend on pressure and volume.

In a differential form the equation of state of an equilibrium gas-liquid mixture is written as follows:

$$\frac{dp_c}{p_c}-\frac{dT_c}{T_c}-\frac{d\gamma_c}{\gamma_c}\frac{1}{1-\frac{\gamma_c}{\gamma_ж(K+1)}}=0. \qquad (1.13)$$

The specific internal energy of a two-phase mixture under the above assumptions is determined by the obvious relationship

$$U_c=c_{v\,c}T_c+U_{c.н}, \qquad (1.14)$$

where $U_{C.H}$ is a certain initial value of the internal energy at temperature $T_{C.H}$. Since in the calculations we are always dealing with an energy difference, then quantity $U_{C.H}$ is eliminated, and for the sake of convenience we can assume that $U_{C.H} = 0$. Then from equations (1.14) and (1.2), if we consider that $U_Г = c_{V\,Г}T_C$ and $U_Ж = c_Ж T_C$, we get

$$U_c=c_{v\,c}T_c=\frac{KU_г+U_ж}{K+1}. \qquad (1.15)$$

The specific enthalpy of the mixture equals:

$$i_c=U_c+\frac{p_c}{J\gamma_c}. \qquad (1.16)$$

Thus, if we use relationship (1.10), (1.12) and (1.15), we find

$$i_c=c_{p\,c}T_c+\frac{p_c}{J\gamma_ж(K+1)}. \qquad (1.17)$$

The dependence of the internal energy of the mixture on temperature has the same form as for an ideal gas, while equation (1.17), which is determined by the enthalpy of the mixture, contains a term which depends on pressure, which we do not have in the analogous equation for an ideal gas. It should be mentioned, however, that in most cases of practical interest the values of the second term in equation (1.17) are very low and can be ignored.

Thus, for example, for water-air mixtures at any value of ratio $G_г/G_ж$ or pressure $p_c \leq 10$ kgf/cm^2 the difference between the actual enthalpy of the mixture and value $c_{p\,c}T_c$ does not exceed 0.3%.

1.2. Isentropic Process of State Change. Speed of Sound in the Mixture

The change in state of the studied two-phase mixture occurs in internal equilibrium, and thus its entropy can be represented as

$$T_c ds_c = dU_c + \frac{1}{J} p_c dv_c. \tag{1.18}$$

If we transform this equation by means of relationships (1.10), (1.13), and (1.15), we get

$$ds_c = c_{vc}\frac{dT_c}{T_c} + \frac{d\left(\frac{1}{\gamma_c}\right)}{\frac{1}{\gamma_c} - \frac{1}{\gamma_ж(K+1)}}\,\frac{R_c}{J}. \tag{1.19}$$

Hence, considering relationship (1.12), we get

$$s_c = c_{vc}\ln T_c\left[\frac{1}{\gamma_c} - \frac{1}{\gamma_ж(K+1)}\right]^{\varkappa_c-1} + s_{c.н} = \frac{R_c}{J}\ln\frac{T_c^{\frac{\varkappa_c}{\varkappa_c-1}}}{p_c} + s_{c.н} + \frac{R_c}{J}\ln R_c, \tag{1.20}$$

where $s_{c.н}$ is the integration constant.

From equation (1.20) it follows that in the case of an isentropic change in state of the mixture p_c, γ_c and T_c are linked by relationships:

$$\frac{p_{c2}}{p_{c1}} = \left(\frac{T_{c2}}{T_{c1}}\right)^{\frac{\varkappa_c}{\varkappa_c-1}}; \tag{1.21}$$

$$\frac{\frac{1}{\gamma_{c1}} - \frac{1}{\gamma_ж(K+1)}}{\frac{1}{\gamma_{c2}} - \frac{1}{\gamma_ж(K+1)}} = \left(\frac{p_{c2}}{p_{c1}}\right)^{\frac{1}{\varkappa_c}} = \left(\frac{T_{c2}}{T_{c1}}\right)^{\frac{1}{\varkappa_c-1}}. \tag{1.22}$$

The speed of sound is determined by equation

$$a^2=(dp/d\varrho)_{s=\mathrm{const}}. \tag{1.23}$$

substitution of which in equation (1.19) provides

$$a_c=\frac{\sqrt{gR_c\varkappa_cT_c}}{1-\dfrac{\gamma_c}{\gamma_{\text{ж}}(K+1)}}=\frac{p_c}{\gamma_c}\sqrt{\frac{g\varkappa_c}{R_cT_c}}. \tag{1.24}$$

Hence it follows that, unlike the ideal gas, the speed of sound in the studied two-phase mixture for the given composition depends not only on temperature, but on specific weight and pressure. As the relative volume of the liquid $V_{\text{ж}}/V_c=\gamma_c/[\gamma_{\text{ж}}(K+1)]$ tends to zero, which for the given composition corresponds to the case of $p_c \to 0$; the speed of sound in the mixture tends to the value corresponding to the speed of sound in an ideal gas, whose adiabatic exponent and gas constant are equal, respectively, to: κ_c and R_c; when $V_{\text{ж}}/V_c \to 1$, when the volume of the gas in the mixture tends to zero, the speed of sound in the mixture rises to infinity.

As an example Fig. 1 shows curves representing the change in the speed of sound in a two-phase water-air mixture as a function of ratio $G_{\text{г}}/G_{\text{ж}}$, calculated from formula (1.24) for several values of pressure at a temperature of T_c = 288°K. Curves $p_c = 0$ corresponds to the speed of sound in an ideal gas when $\kappa_{\text{г}} = \kappa_c$ and $R_{\text{г}} = R_c$.

If we examine Fig. 1 we see that when $p_c = 0$ with a decreased gas content the speed of sound in the mixture decreases steadily from 340 m/s when $K = 10^3$ to [illegible] m/s when $K = 10^{-2}$ and continues to decrease with a further drop in $K = G_{\text{г}}/G_{\text{ж}}$. This is explained by the fact that with a decrease in K the adiabatic exponent of the mixture tends to unity, while R_c tends to zero. In a range of change of coefficient K from 10^3 to 10^{-1} and mixture pressure from zero to 5 kgf/cm^2 the speed of sound in the mixture practically

coincides with the speed of sound in an ideal gas (curve $p_c = 0$). With an increase in pressure when K = const the speed of sound in the mixture rises steadily. Thus, for example, when K = 0.01 with an increase in pressure from zero to 10 and to 100 kgf/cm^2 the quantity a_c rises accordingly from 28 m/s to 63 and 360 m/s. Curves $a_c(K)$ when p_c = const and T_c = const have a minimum, which with an increase in pressure moves toward greater values of K.

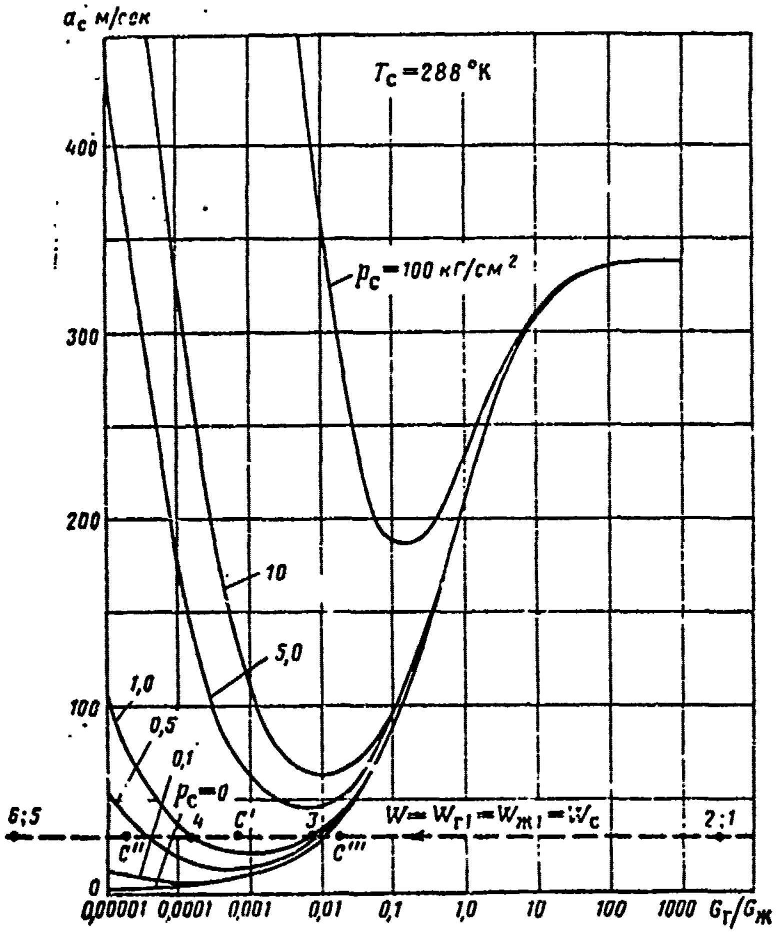

Fig. 1. Curves representing change in speed of sound in a two-phase water-air mixture depending on ratio $G_г/G_ж$.
Designations: м/сек = m/s; кГ/см^2 = kgf/cm^2.

1.3. Calculating One-Dimensional Flows of a Mixture Without Losses

Let us examine the established flow of the equilibrium two-phase mixture in a stationary channel of variable suction with heat-insulated walls, which was described above. According to the assumption of mechanical equilibrium for the components of the mixture we will consider the velocity of the gas and the liquid in any section of the channel to be equal. Then, for the sake of simplicity, we will assume that there are no losses in total pressure of the mixture as it flows in the channel.

1.3.1. General Case of the Flow

The equation of the conservation of energy for the studied flow is written, as we know, in the form of

$$\frac{W_c^2}{2gJ}+i_c=\text{const}, \tag{1.25}$$

from which, if we consider relationship (1.17) and introduce stagnation parameters, we get

$$\frac{W_c^2}{2gJ}+c_{pc}T_c+\frac{p_c}{J\gamma_ж(K+1)}=c_{pc}T_{c0}+\frac{p_{c0}}{J\gamma_ж(K+1)}=i_{c0}. \tag{1.26}$$

For assigned flow parameters in the studied channel section equation (1.26) lets us find the stagnation enthalpy of the mixture. To determine p_{c0} and T_{c0}, in addition to equation (1.26), we must use the condition of isentropicity ($s_{c0} = s_c$), which, as follows from relationship (1.21), gives us

$$\frac{p_{c0}}{p_c}=\left(\frac{T_{c0}}{T_c}\right)^{\frac{\varkappa_c}{\varkappa_c-1}}. \tag{1.27}$$

If we substitute relationship (1.27) in (1.26), we get the equation

$$c_{pc}T_c\left(\frac{p_{c0}}{p_c}\right)^{\frac{\varkappa_c-1}{\varkappa_c}}+\frac{p_c}{J\gamma_ж(K+1)}\frac{p_{c0}}{p_c}=i_{c0}. \tag{1.28}$$

whose solution provides the thought value p_{c0}; T_{c0} is then found from relationship (1.27).

From equation (1.26) it follows that as the two-phase mixture flows in a stationary channel with heat-insulated walls, when the stagnation enthalpy remains invariable, the stagnation temperature can change. Quantity T_{c0} when i_{c0} = const remains unchanged only in the case of an isentropic flow, when total pressure remains unchanged.

Let us find the parameters of state in a critical section of the channel, in which the velocity of the flow is equal to the local speed of sound. If we assume that $W_c = a_c = a_{c.\kappa}$, then from equation (1.26), using relationships (1.10), (1.24), and (1.27), we get the following equation for determining static pressure $p_{c.\kappa}$ in the critical section:

$$\frac{\varkappa_c}{2R_cT_{c0}}\left(\frac{p_{c0}}{p_{c.\kappa}}\right)^{\frac{\varkappa_c-1}{\varkappa_c}}\left[R_cT_{c0}\left(\frac{p_{c.\kappa}}{p_{c0}}\right)^{\frac{\varkappa_c-1}{\varkappa_c}}+\frac{p_{c.\kappa}}{p_{c0}}\frac{p_{c0}}{\gamma_{ж}(K+1)}\right]^2+$$
$$+Jc_{pc}T_{c0}\left(\frac{p_{c.\kappa}}{p_{c0}}\right)^{\frac{\varkappa_c-1}{\varkappa_c}}+\frac{p_{c.\kappa}}{p_{c0}}\frac{p_{c0}}{\gamma_{ж}(K+1)}=Ji_{c0}. \qquad (1.29)$$

After determining $p_{c.\kappa}$, from relationship (1.27) we find $T_{c.\kappa}$, and then we determine $W_{c.\kappa} = a_{c.\kappa}$ according to formula (1.24). From equation (1.29) it follows that for assigned physical properties in a two-phase mixture critical drops in pressures $(p_{c.\kappa}/p_{c0})$ and temperatures $(T_{c.\kappa}/T_{c0})$ depend on the stagnation parameters T_{c0}, p_{c0}.

In conclusion let us give the calculation order for the isentropic flow of the studied gas-liquid mixture for several cases where the original parameters, encountered in studying ejector characteristics, are given. The physical constants of the mixture will in this case be considered known.

I. The parameters of state and the velocity of the mixture in channel section 1 with section area f_1 are given. We find the parameters of state and the velocity of the mixture in channel section 2 with cross-section area f_2.

1) From relationships (1.26), (1.27), and (1.28), we find the stagnation parameters of the mixture (i_{c0}, p_{c0}, T_{c0}).

2) If we solve equation

$$\left(\frac{W_{c1}\gamma_{c1}f_{c1}}{f_{c2}}\right)^2\left[\frac{R_cT_{c0}}{p_{c0}}\left(\frac{p_{c0}}{p_{c2}}\right)^{\frac{1}{\varkappa_c}}+\frac{1}{\gamma_{ж}(K-1)}\right]^2=$$
$$=2gJc_{pc}T_{c0}\left[1-\left(\frac{p_{c2}}{p_{c0}}\right)^{\frac{\varkappa_c-1}{\varkappa_c}}\right]+\frac{2gp_{c0}}{\gamma_{ж}(K-1)}\left(1-\frac{p_{c2}}{p_{c0}}\right), \tag{1.30}$$

which is obtained from equation (1.26) using the equation of the conservation of mass

$$\gamma_{c2}f_{c2}W_{c2}=\gamma_{c1}f_{c1}W_{c1}, \tag{1.31}$$

we find static pressure p_{c2}.

3) From formulas (1.27), (1.10), (1.31), and (1.24) we calculate T_{c2}, γ_{c2}, W_{c2}, and a_{c2}, after which we determine the number $M_{c2} = W_{c2}/a_{c2}$.

Equation (1.30) gives us two values for pressure p_{c2}. The lesser of these values corresponds to the supersonic flow of the mixture, the greater - to the subsonic. In a case where the area of the cross section of the channel either rises steadily or decreases steadily, the supersonic root is realized only when $M_{c1} \geq 1$, and the subsonic - when $M_{c1} \leq 1$.

II. The stagnation parameters of the mixture and the area of the outlet section of the contracted channel are given. We find the dependence of the flow rate, the parameters of state, and the outflow velocity for the mixture on pressure at the edge of the channel.

1) If we solve equation (1.29) we find $p_{c.\kappa}$, which corresponds to the shut-off regime of the channel, after which we find $T_{c.\kappa}$, $\gamma_{c.\kappa}$, and $W_{c.\kappa} = a_{c.\kappa}$ from relationships (1.27), (1.10) and (1.24).

2) We assign a series of values p_c in a range from $p_{c.\kappa}$ to p_{c0} and from formulas (1.27) and (1.10) we find the values of T_c and γ_c which correspond to them.

3) Let us find the outflow velocity of the mixture

$$W_c = \sqrt{2gJc_{pc}T_{c0}\left(1-\frac{T_c}{T_{c0}}\right)+\frac{2gp_{c0}}{\gamma_{\kappa}(K+1)}\left(1-\frac{p_c}{p_{c0}}\right)}, \qquad (1.32)$$

after which we determine the flow rate of the mixture and number M_c.

III. We assign the stagnation parameters of the mixture and the areas of the critical and outlet sections of the supersonic nozzle. We find the parameters of state and the velocity of the mixture in the outlet section of the nozzle under its rated operational mode.

1) According to point (1) of problem II we find the quantities $p_{c.\kappa}$, $T_{c.\kappa}$, $\gamma_{c.\kappa}$, and $W_{c.\kappa}$.

2) Further calculation is according to points (2) and (3) of problem I.

1.3.2. Liquid Volume in Mixture is Small in Comparison to Gas Volume

If the volume of the liquid in the mixture is negligibly small as compared to the gas volume, then the equation of state (1.10) is reduced to the same form that it has for an ideal gas:

$$p_c = R_c \gamma_c T_c. \qquad (1.33)$$

The same thing happens with equations (1.17), (1.21), (1.22), and (1.24), which are written as follows:

$$i_c = c_{pc} T_c; \qquad (1.34)$$

$$\frac{p_{c2}}{p_{c1}} = \left(\frac{\gamma_{c2}}{\gamma_{c1}}\right)^{\varkappa_c} = \left(\frac{T_{c2}}{T_c}\right)^{\frac{\varkappa_c}{\varkappa_c - 1}}; \qquad (1.35)$$

$$a_c = \sqrt{g \ldots} \qquad (1.36)$$

Equation (1.26) is reduce to the form

$$\frac{W_c^2}{2gJ} + c_{pc} T_c = c_{pc} T_{c0} = i_{c0} \qquad (1.37)$$

from which, using (1.36), we find the critical speed of sound

$$a_{c.\kappa} = \sqrt{\frac{2(\varkappa_c - 1)}{\varkappa_c + 1} gJc_{pc} T_{c0}} = \sqrt{\frac{2\varkappa_c}{\varkappa_c + 1} gR_c T_{c0}}. \qquad (1.38)$$

If we introduce the reduced velocity $\lambda_c = W_c / a_{c.\kappa}$, then from the relationships of (1.37), using formulas (1.33) and (1.35), we get the known gas-dynamics functions:

$$\left.\begin{aligned} T(\lambda_c) &= \frac{T_c}{T_{c0}} = 1 - \frac{\varkappa_c - 1}{\varkappa_c + 1} \lambda_c^2; \\ \varrho(\lambda_c) &= \frac{\varrho_c}{\varrho_{c0}} = \left(1 - \frac{\varkappa_c - 1}{\varkappa_c + 1} \lambda_c^2\right)^{\frac{1}{\varkappa_c - 1}}; \\ p(\lambda_c) &= \frac{p_c}{p_{c0}} = \left(1 - \frac{\varkappa_c - 1}{\varkappa_c + 1} \lambda_c^2\right)^{\frac{\varkappa_c}{\varkappa_c - 1}}, \end{aligned}\right\} \qquad (1.39)$$

which, together with functions

$$q(\lambda_c)=\lambda_c\left(1-\frac{\varkappa_c-1}{\varkappa_c+1}\lambda_c^2\right)^{\frac{1}{\varkappa_c-1}}; \tag{1.40}$$

$$z(\lambda_c)=\lambda_c+\frac{1}{\lambda_c}, \tag{1.41}$$

enable us to greatly simplify the calculations. The quantity λ_c, which is contained in these equations, can vary from zero to $\lambda_{c\,max}=\sqrt{\frac{\varkappa_c+1}{\varkappa_c-1}}$. When $\lambda_c = 1$ from equations (1.39)-(1.41) we get

$$\left.\begin{aligned} &T_c(1)=\frac{2}{\varkappa_c+1}; \quad \varrho_c(1)=\left(\frac{2}{\varkappa_c+1}\right)^{\frac{1}{\varkappa_c-1}};\\ &p_c(1)=\left(\frac{2}{\varkappa_c+1}\right)^{\frac{\varkappa_c}{\varkappa_c-1}}; \quad q_c(1)=\varrho_c(1); \quad z_c(1)=2,\ldots \end{aligned}\right\} \tag{1.42}$$

1.3.3. The Amount of Liquid in the Mixture is Very Great

This case is characteristic for a vacuum liquid-gas ejector, in which the mass flow rate exceeds the mass flow rate of the gas ($K = 10^{-3}$-10^{-5}) by several orders. In this case the temperature of the mixture as it flows in a channel of variable section remains virtually unchanged, and thus the flow can be regarded approximately as isothermic ($T_c = T_{c0}$ = const).

The equation of state (1.10) of a two-phase mixture can be written for the studied case as follows:

$$p_c\left(v_c-\frac{v_{\text{ж}}}{K+1}\right)=\frac{p_c}{\gamma_c}\left[1-\frac{\gamma_c}{\gamma_{\text{ж}}(K+1)}\right]=b, \tag{1.43}$$

where

$$b=R_cT_0=\text{const}. \tag{1.44}$$

For the speed of sound we obtain, expression (1.24) under condition T_c = const the following expression:

$$a_c=\sqrt{\frac{g}{b}\frac{p_c}{\gamma_c}}. \qquad (1.45)$$

According to the first law of thermodynamics, heat $q_{1,2}$, which is conducted to the flow through the wall of the channel in the segment between sections 1-2, equals:

$$q_{1,2}=U_{c2}-U_{c1}+\frac{W_{c2}^2-W_{c1}^2}{2gJ}+\frac{1}{J}(p_{c2}v_{c2}-p_{c1}v_{c1}). \qquad (1.46)$$

In the case of an isothermic flow $T_{c2} = T_{c1}$, and thus according to relationship (1.15) we have

$$U_{c2}-U_{c1}=0. \qquad (1.47)$$

If we use equation (1.43), then we get

$$\frac{1}{J}(p_{c2}v_{c2}-p_{c1}v_{c1})=\frac{p_{c2}-p_{c1}}{J\gamma_{\text{ж}}(K+1)}. \qquad (1.48)$$

All of the heat which is conducted to the working body in the studied isothermic process is spent in producing work, and thus we can write

$$q_{1,2}=\frac{1}{J}\int_1^2 p_c dv_c,$$

from which, using equation (1.43), we get

$$q_{1,2}=\frac{b}{J}\ln\frac{p_{c1}}{p_{c2}}. \qquad (1.49)$$

If we consider expressions (1.47), (1.48), and (1.49), and the fact that according to our assumption there are no losses in total pressure, we reduce the equation of the conservation of energy (1.46) to the following form:

$$\frac{W_{c1}^2}{2g}+b\ln p_{c1}+\frac{p_{c1}}{\gamma_{ж}(K+1)}=\frac{W_{c2}^2}{2g}+b\ln p_{c2}+$$
$$+\frac{p_{c2}}{\gamma_{ж}(K+1)}=b\ln p_{c0}+\frac{p_{c0}}{\gamma_{ж}(K+1)}=\text{const.} \tag{1.50}$$

If the velocity values and the parameters of state are assigned in a certain section of the channel, equation (1.50) will let us find the total pressure of the mixture. Quantity γ_{c0} is then determined according to formula (1.43).

The critical speed of sound $a_{c.к}$ in an isothermic flow is found from equation (1.50), which when $W_c = a_c = a_{c.к}$ is reduced, using relationships (1.43) and (1.45), to the form of

$$\frac{a_{c.к}^2}{2g}+b\ln\left[\gamma_{ж}(K+1)\left(\sqrt{\frac{b}{g}}\,a_{c.к}-b\right)\right]+$$
$$+\left(\sqrt{\frac{b}{g}}\,a_{c.к}-b\right)=b\ln p_{c0}+\frac{p_{c0}}{\gamma_{ж}(K+1)}. \tag{1.51}$$

After we have determined the critical velocity, then from expression (1.45) we find the ratio $p_{c.к}/\gamma_{c.к}$, and then from equation (1.43) we determine the static pressure and the specific weight of the mixture in the critical section of the channel.

From equation (1.50) we find the following expression for determining the outflow velocity of the mixture from the nozzle at an assigned ratio of values p_c/p_{c0}:

$$W_c=\sqrt{2g\left[b\ln\frac{p_{c0}}{p_c}+\frac{p_{c0}}{\gamma_{ж}(K+1)}\left(1-\frac{p_c}{p_{c0}}\right)\right]}. \tag{1.52}$$

Let us assign the calculation order of an isothermic flow of the mixture in a channel without losses.

Let the physical constants, the parameters of state, and the velocity of the mixture in section 1 of the channel with section area f_1 be assigned. We find the flow parameters of the mixture in section 2 of the channel with section area f_2.

1) If we solve equation

$$\frac{1}{2g}\left\{\frac{\gamma_{c1} W_{c1} f_{c1}}{p_{c2} f_{c2}}\left[b + \frac{p_{c2}}{\gamma_{ж}(K+1)}\right]\right\}^2 + b \ln p_{c2} + \frac{p_{c2}}{\gamma_{ж}(K+1)} = \frac{W_{c1}^2}{2g} + b \ln p_{c1} + \frac{p_{c1}}{\gamma_{ж}(K+1)}, \qquad (1.53)$$

then we find static pressure p_{c2}.

2) From equation (1.43) we find the specific weight of the mixture γ_{c2}.

3) From equation (1.31) we determine the velocity of the mixture W_{c2}.

1.4. Relationships for a Plane Shock

Let us assign the relationships which determine the parameters of the flow for a plane shock developing in a supersonic two-phase flow mixture. Subscripts "1" and "2" denote flow parameters in front of and behind the shock, respectively.

1.4.1. General Case of the Flow

From the equation of continuity, considering that $f_{c1} = f_{c2} = f_c$, we have

$$\rho_{c2} = \frac{m}{W_{c2}}, \qquad (1.54)$$

where $m = \rho_{c1} W_{c1}$ is the per-second mass flow rate of the mixture through a unit of the area.

The equation of momentum for sections 1-2 gives us

$$p_{c2}=h-mW_{c2}, \tag{1.55}$$

where $h = mW_{c1} + p_{c1}$. If we use relationships (1.54) and (1.55), then from equation (1.10) we find

$$T_{c2}=\frac{h-mW_{c2}}{gR_c}\left[\frac{W_{c2}}{m}-\frac{1}{\varrho_{ж}(K+1)}\right]. \tag{1.56}$$

If from the equation of conservation of energy (1.26) we exclude quantities p_{c2} and T_{c2}, then using expressions (1.55) and (1.56), we get

$$\frac{\varkappa_c+1}{2}W_{c2}^2-\left[\frac{\varkappa_c h}{m}+\frac{m}{\varrho_{ж}(K+1)}\right]W_{c2}+\frac{h}{\varrho_{ж}(K+1)}+(\varkappa_c-1)gJi_{c0}=0, \tag{1.57}$$

from which we obtain

$$W_{c2}=\frac{1}{\varkappa_c+1}\left\{\frac{\varkappa_c h}{m}+\frac{m}{\varrho_{ж}(K+1)}\pm\right.$$
$$\left.\pm\sqrt{\left[\frac{\varkappa_c h}{m}+\frac{m}{\varrho_{ж}(K+1)}\right]^2-2(\varkappa_c+1)\left[\frac{h}{\varrho_{ж}(K+1)}+(\varkappa_c-1)gJi_{c0}\right]}\right\}. \tag{1.58}$$

After we have determined the velocity of the mixture W_{c2} behind the plane shock, then from relationships (1.54), (1.55), and (1.10), we find the values of ρ_{c2}, p_{c2}, and T_{c2}. Then, from equation (1.24) we find the speed of sound a_{c2} and number $M_{c2} = W_{c2}/a_{c2}$.

Stagnation parameters T_{c02} and p_{c02} behind the shock are found from equations (1.27) and (1.28) from known values of p_{c2}, T_{c2}, and W_{c2}.

1.4.2. Case of a Relatively Large Gas Content in Mixture

In the case where the volume of the liquid in the mixture is very small, the two-phase gas-liquid mixture can be regarded as

an ideal gas. The parameters of the flow beyond the plane shock determined in this case by known relationships:

$$\lambda_{c2}\lambda_{c1}=1; \tag{1.59}$$

$$p_{c02}=p_{c01}\frac{q(\lambda_{c1})}{q(\lambda_{c2})}; \tag{1.60}$$

$$T_{c02}=T_{c01}. \tag{1.61}$$

1.4.3. Case of Isothermic Flow

This case was examined thoroughly in [18].

From the equation of the conservation of mass we have

$$\gamma_{c2}=\gamma_{c1}\frac{W_{c2}}{W_{c1}}, \tag{1.62}$$

from which, if we use expression (1.45), we find

$$p_{c2}=p_{c1}\frac{M_{c1}}{M_{c2}}, \tag{1.63}$$

where

$$M_c=W_c/a_c.$$

If we transform the equation of momentum, which is written in the form of

$$\varrho_{c2}W_{c2}^2-\varrho_{c1}W_{c1}^2=p_{c1}-p_{c2}, \tag{1.64}$$

by means of expressions (1.45) and (1.63), then by introducing the number M_c, we get

$$\frac{M_{c1}M_{c2}}{b}\frac{p_{c2}}{\gamma_{c2}}-\frac{M_{c1}^2}{b}\frac{p_{c1}}{\gamma_{c1}}=1-\frac{M_{c1}}{M_{c2}},$$

from which, after simple transformations, we get

$$M_{c2}M_{c1}=1. \qquad (1.65)$$

If we substitute this expression in relationship (1.63), we find

$$p_{c2}=p_{c1}M_{c1}^{2}. \qquad (1.66)$$

From expression (1.43), using (1.66), we get

$$\gamma_{c2}=\frac{\gamma_{ж}(K+1)\,p_{c1}M_{c1}^{2}}{b\gamma_{ж}(K+1)+p_{c1}M_{c1}^{2}}. \qquad (1.67)$$

The speed of sound beyond the plane shock, as easily demonstrated, equals:

$$a_{c2}=a_{c1}\frac{p_{c1}M_{c1}^{2}+b\gamma_{ж}(K+1)}{p_{c1}+b\gamma_{ж}(K+1)}. \qquad (1.68)$$

In conclusion we note that all relationships obtained in this chapter will also be correct for the flow of a two-phase gas mixture and for rather small solid particles.

CHAPTER 2

EJECTION EQUATIONS

2.1. Ejector System and Basic Assumptions

The studied ejector system is shown in Fig. 2.

The gas is supplied to the mixing chamber through a convergent subsonic or a divergent supersonic (dashed lines) nozzle, the liquid - through a convergent nozzle.[1] In the mixing chamber, which is cylindrical in shape, a homogeneous two-phase gas-liquid mixture is formed, which flows out of the ejector through a subsonic divergent diffuser or a supersonic diffuser with a throat (dashed lines). The mutual arrangement, the number, and the shape of the cross sections of the nozzles can vary.

The flow parameters of the gas and the liquid in the inlet section of the mixing chamber, which coincides with the outlet sections of the nozzles, are denoted by the subscript "1," the parameters of the mixture in the outlet sections of the mixing

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[1]The liquid nozzle in a case where pressure on its edge in a rated operational ejector regime is less than saturation pressure p_{3H}, can also be a divergent-supersonic nozzle.

chamber and the diffuser - by subscripts "3" and "4." Subscripts "2" and "к" denote the parameters of the gas and liquid jets in the shut-off section of the mixing chamber and in the critical section of the supersonic gas nozzle, respectively.

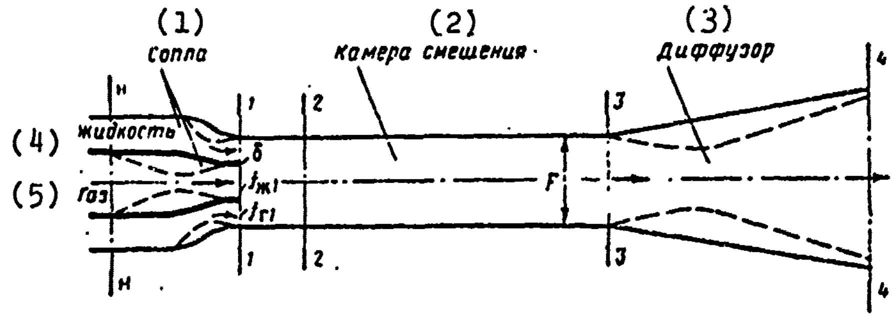


Fig. 2. Ejector system.
KEY: (1) Nozzles, (2) Mixing chamber; (3) Diffuser; (4) Liquid; (5) Gas.

As the basic geometrical ejector parameters we use: coefficient α, which is equal to the ratio of areas of the outlet sections of the gas ($f_{\Gamma 1}$ in m^2) and liquid ($f_{ж1}$ in m^2) nozzles ($\alpha = f_{\Gamma 1}/f_{ж1}$); coefficient $\bar{\delta}$, which is equal to the ratio of the total end area of the nozzles (δ in m^2) to the area of the cross section of the mixing chamber (F in m^2) ($\bar{\delta} = \delta/F$), and the relative area of the critical section of the supersonic nozzle ($\tilde{f}_{\Gamma.к} = f_{\Gamma.к}/f_{\Gamma 1}$).

Losses in the nozzles and the diffuser of the ejector are described by the values of the pressure recovery coefficient

$$\nu_{\Gamma 1н}=\frac{p_{\Gamma 01}}{p_{\Gamma 0н}};\quad \nu_{ж1н}=\frac{p_{ж01}}{p_{ж0н}};\quad \nu_{c4,3}=\frac{p_{c04}}{p_{c03}}. \tag{2.1}$$

In deriving the ejection equations we make the following assumptions and limitations: 1) the walls of the nozzles, the mixing chamber, and the diffuser do not conduct heat; 2) there is no friction on the wall of the mixing chamber; 3) the flow in the nozzles and diffuser is one-dimensional; 4) the mixing process is mechanical and is not accompanied by chemical conversion; 5) the gas is ideal and does not dissolve in the liquid and its

physical constants ($c_{p\,г}$, $c_{v\,г}$, $\kappa_{г}$) do not depend on temperature or pressure; 6) the heat capacity and the specific weight of the liquid in the mixture do not depend on temperature or pressure; 7) in the outlet section of the mixing chamber a thermodynamically and mechanically equilibrium gas-vapor-liquid mixture is formed, in which the vapor content is negligibly small; 8) the ejector axis is horizontal.

It should be mentioned that the absence of vapor is assumed only for the flow of the mixture in the outlet segment of the mixing chamber and in the ejector diffuser. In the initial segment of the mixing chamber partial evaporation of the liquid is possible; the flow of liquid in the nozzle can be accompanied by evaporation.

2.2. Deriving Ejection Equations

In order to determine the parameters of state and the velocity of the mixture in the outlet section of the mixing chamber we solve the equations of the conservation of mass, momentum, and energy together, assuming in this case that the parameters of the velocity of the gas and the liquid in the inlet section of the mixing chamber are known.

The inlet section of the mixing chamber coincides with the outlet sections of the nozzles, and thus

$$f_{г1}+f_{ж1}+\delta=F. \tag{2.2}$$

If we introduce the geometrical parameter of the ejector α, then from equation (2.2) we get:

$$\bar{f}_{ж1}=\frac{f_{ж1}}{F}=\frac{1-\bar{\delta}}{\alpha+1}; \tag{2.3}$$

$$\bar{f}_{г1}=\frac{f_{г1}}{F}=\frac{\alpha(1-\bar{\delta})}{\alpha+1}. \tag{2.4}$$

From the equation of the conservation of mass for the inlet and outlet section of the mixing chamber

$$G_{c3}=G_{г.п}+G_{ж.н}=G_{ж.п}(K+1) \qquad (2.5)$$

we get

$$\gamma_{c3}=\frac{G_{ж.н}(K+1)}{fW_{c3}}. \qquad (2.6)$$

In the general case the flow of liquid in the nozzle can be accompanied by evaporation. In this case a moist vapor flows from the nozzle. This is a two-phase vapor-liquid mixture, whose state parameters and velocity are denoted by the subscript "п.ж." The amount of steam in this mixture is described by $x = G_п/G_{ж.п} = G_п/(G_п + G_ж)$, which is called the degree of dryness or the vapor content. When $x_1 = 0$ a liquid flows from the nozzle whose parameters are marked by the subscript "ж," and when $x = 1$ - dry saturated or superheated vapor, whose parameters are marked as "п." The ejection equations obtained below are valid for all of these cases, and thus to abbreviate the writing the parameters of the stream at the edge of the liquid nozzle are indicated by "п.ж."

From equation (2.6), if we use the relationship (2.3) and the obvious relationship

$$G_{ж.п}=\gamma_{п.ж1}W_{п.ж1}f_{п.ж1}, \qquad (2.7)$$

we have

$$\gamma_{c3}=\frac{(1-\delta)(K+1)\,\gamma_{п.ж1}W_{п.ж1}}{(a+1)W_{c3}}. \qquad (2.8)$$

The per-second flow rate of the gas equals

$$G_{г.н}=\frac{p_{г01}q(\lambda_{г1})\,a_{к.г.н}f_{г1}}{R_гT_{г0н}}. \qquad (2.9)$$

If we use expressions (2.7) and (2.9), we get

$$K=\frac{G_{г.н}}{G_{ж.н}}=\frac{\alpha p_{г01}a_{к.г.н}q(\lambda_{г1})}{R_гT_{г0н}\gamma_{п.ж1}W_{п.ж1}}=\frac{\alpha p_{г01}q(\lambda_{г1})}{\gamma_{п.ж1}W_{п.ж1}}\sqrt{\frac{2g\varkappa_г}{(\varkappa_г+1)R_гT_{г0н}}}, \qquad (2.10)$$

where

$$p_{г01}=p_{г0н}\nu_{г1н}.$$

The equation of momentum for the inlet and outlet sections of the mixing chamber can be represented in the form of

$$W_{с3}+\frac{p_{с3}Fg}{G_{с3}}=\frac{K}{K+1}\left(W_{г1}+\frac{p_{г1}f_{г1}g}{G_{г.н}}\right)+$$
$$+\frac{1}{K+1}\left(W_{п.ж1}+\frac{p_{п.ж1}f_{ж1}g}{G_{ж.н}}\right)+\frac{1}{K+1}\,\frac{gp_1\bar{\delta}(\alpha+1)}{\gamma_{п.ж1}W_{п.ж1}(1-\bar{\delta})}. \qquad (2.11)$$

where p_1 is pressure at the end of the nozzles.

If we introduce the designations

$$n_{г1}=W_{г1}+\frac{p_{г1}f_{г1}g}{G_{г.н}}=\frac{\varkappa_г+1}{2\varkappa_г}a_{к.г.н}z(\lambda_{г1}); \qquad (2.12)$$

$$n_{п.ж1}=W_{п.ж1}+\frac{p_{п.ж1}g}{\gamma_{п.ж1}W_{п.ж1}}; \qquad (2.13)$$

$$n_1=\frac{gp_1\bar{\delta}}{G_{ж.н}}=\frac{\bar{\delta}(\alpha+1)}{1-\bar{\delta}}\,\frac{gp_1}{\gamma_{п.ж1}W_{п.ж1}} \qquad (2.14)$$

and solve equation (2.11) with respect to the pressure of the mixture, we get

$$p_{с3}=\frac{(1-\bar{\delta})\gamma_{п.ж1}W_{п.ж1}}{g(\alpha+1)}[Kn_{г1}+n_{п.ж1}+n_1-(K+1)W_{с3}]. \qquad (2.15)$$

Static pressure p_1, which is contained in (2.14), and represents static pressure in the stagnant region formed near the dull edges of the nozzles, can be assumed in the first approximation to be equal to the static pressure at the edge of the gas nozzles in a case where $\lambda_{г1} < 1$, or equal to the pressure at the edge of the liquid nozzle when $\lambda_{г1} > 1$, while $p_{ж1} < p_{s1}$. The total pressure of the liquid at the edge of the nozzle is equal to:

$$p_{ж01} = p_{ж0н} \nu_{ж1н}.$$

The temperature of the mixture T_{c3} is found by using equations (2.8) and (2.15) from the equation of state for a two-phase gas-liquid mixture (1.10):

$$T_{c3} = \frac{1}{gR_c}\left\{-W_{c3}^2 + \frac{[(\alpha+1)(Kn_{г1} + n_{п.ж1} + n_1) + (K+1)(1-\bar{\varepsilon})W_{п.ж1}]W_{c3}}{(K+1)(\alpha+1)} - \frac{(Kn_{г1} + n_{п.ж1} + n_1)(1-\bar{\varepsilon})W_{п.ж1}}{(K+1)(\alpha+1)}\right\}. \qquad (2.16)$$

To determine with the help of the equations (2.8), (2.15), and (2.16) the parameters of state of the mixture from known parameters of the gas and fluid at the inlet to the mixing chamber we must know the velocity of the mixture W_{c3}. Quantity W_{c3} is found from the equation of the conservation of energy. This equation, with equation (1.26) considered, can be written in the form of

$$\frac{W_{c3}^2}{2} + gJc_{pc}T_{c3} + \frac{gp_{c3}}{\gamma_ж(K+1)} = \frac{gJ}{K+1}(Ki_{г0н} + i_{ж0н}) = gJi_{c03}, \qquad (2.17)$$

where

$$i_{г0н} = c_{pг}T_{г0н}; \quad i_{ж0н} = U_{ж0н} + \frac{p_{ж0н}}{J\gamma_{ж.н}} = c_ж T_{ж.н} + \frac{p_{ж0н}}{J\gamma_{ж.н}}.$$

If we use the relationships of (2.15) and (2.16), then from equation (2.17) we get

$$W_{c3} = l \pm \sqrt{l^2 - 2d}. \tag{2.18}$$

where

$$l = \frac{\varkappa_c (\alpha + 1)(K n_{г1} + n_{п.ж1} + n_1) + (1 - \bar{\delta})(K - 1) W_{п.ж1}}{(\varkappa_c + 1)(\alpha + 1)(K + 1)}; \tag{2.19}$$

$$d = \frac{(1 - \bar{\delta})(K n_{г1} + n_{п.ж1} + n_1) W_{п.ж1}}{(\varkappa_c + 1)(\alpha + 1)(K + 1)} + \frac{\varkappa_c - 1}{\varkappa_c + 1} gJ i_{c0}, \tag{2.20}$$

while the quantities κ_c, K, $n_{г1}$, $n_{п,ж1}$, n_1, and i_{c0} are determined from (1.3), (2.10), (2.12), (2.13), (2.14), and (2.17).

Formula (2.18) gives two velocity values for the mixture in the outlet section of the mixing chamber at assigned parameters of state and velocity of the gas and the liquid at the edge of the nozzle. The lower of these values corresponds to the subsonic ($W''_{c3} < a''_{c3}$), the greater - to the supersonic ($W'_{c3} > a'_{c3}$) flow of the mixture. In a case where the radicand expression in formula (2.18) reverts to zero, the velocity of the mixture is equal to the critical speed of sound ($W_{c3} = a_{c3} = a_{к,c3}$). In the following chapter we will discuss the problem of when a certain flow is realized.

2.3. Approximate Calculation of Mixture Parameters

2.3.1. Volume of Liquid in the Mixture is Negligibly Small

In cases where the volume of the liquid in the mixture is small in comparison to the volume of gas, the mixture, as we have already indicated, can be regarded as an ideal gas with physical constants $C_{p\,c}$, $C_{v\,c}$, κ_c, R_c, which are determined according to formulas (1.1), (1.2), (1.3), (1.9). The equation of state of

the mixture (1.10) is in this case reduced to the form of (1.33), while the heat content and the speed of sound are determined by (1.34) and (1.36).

Here the critical speed of sound is written in the form of (1.33), where the static parameters of the mixture are bound to the stagnation parameters of the relationship of (1.39).

The parameters of the mixture at the ejector outlet are in this case calculated as follows.

1) By means of relationships (1.1), (1.3), (1.9), (2.10), (2.12), (2.13), (2.14) and (2.17) we find the quantities $c_{p\ c}$, κ_c, R_c, K, $n_{г1}$, $n_{п.ж1}$, n_1 and i_{c03}.

2) We find the stagnation temperature

$$T_{c03} = \frac{i_{c03}}{c_{pc}} = \frac{1}{c_{pc}(K+1)} [K c_{pг} T_{г0н} + i_{ж0н}] \tag{2.21}$$

and the critical velocity of the mixture.

3) We determine the reduced velocity of the mixture from the equation of momentum (2.11), which is reduced to the form of

$$z(\lambda_{c3}) = \frac{2\varkappa_c}{\varkappa_c+1} \frac{K n_{г1} + n_{п.ж1} + n_1}{a_{к.c3}(K+1)} = D, \tag{2.22}$$

from which we get

$$\lambda_{c3} = \frac{1}{2}(D \pm \sqrt{D^2-4}). \tag{2.23}$$

The plus sign in front of the root gives us the supersonic velocity value ($\lambda'_{c3} > 1$), and the minus sign - the subsonic ($\lambda''_{c3} < 1$) velocity value of the mixture.

4) From the equation of the conservation of mass (2.5), by introducing the reduced velocity of the mixture and the stagnation parameters, we find the two values of total pressure of the mixture (p'_{c03} and p''_{c03}) at the outlet from the mixing chamber.

$$p_{c03}=\frac{(1-\bar{\delta})(K+1)\gamma_{n.\text{ж}1}W_{n.\text{ж}1}R_cT_{c03}}{(\alpha+1)a_{\text{к}.c3}q(\lambda_{c3})}=$$
$$=\frac{(1-\bar{\delta})(K+1)\gamma_{n.\text{ж}1}W_{n.\text{ж}1}}{(\alpha+1)q(\lambda_{c3})}\sqrt{\frac{(\varkappa_c-1)R_cT_{c03}}{2g\varkappa_c}} \qquad (2.24)$$

and at the outlet from the diffuser

$$p_{c04}=\nu_{4,3}p_{c03}.$$

5) Using the relationships of (1.39) we find T'_{c3}, T''_{c3}, γ'_{c3}, γ''_{c3}, p'_{c3}, p''_{c3}, and then the velocity values of the mixture W'_{c3} and W''_{c3}, which correspond to the supersonic and the subsonic flows of the mixture.

2.3.2. The Part by Weight of the Gas in the Mixture is Low

In this case the flow of the mixture can be regarded as isothermic. The temperature of the mixture for pressures which are not too great can be fround from equation

$$T_{c3}=T_{c03}=\frac{i_{c03}}{c_{pc}}=\frac{1}{c_{pc}(K+1)}[Kc_{p\text{г}}T_{\text{г}0\text{н}}+i_{\text{ж}0\text{н}}]; \qquad (2.25)$$

if, however, $T_{\text{г}0\text{н}}$ and $T_{\text{ж.н}}$ differ only slightly, then we can assume that $T_{c3}=T_{c03}=T_{\text{ж.н}}$.

If from equation (2.15) we exclude p_{c3} by using expressions (1.10) and (2.8), then after simple transformations we get

$$W_{c3}^2-\left[\frac{N_1}{K+1}+\frac{(1-\bar{\delta})W_{n.\text{ж}1}}{\alpha+1}\right]W_{c3}+\frac{(1-\bar{\delta})N_1W_{n.\text{ж}1}}{(\alpha+1)(K+1)}+gb=0, \qquad (2.26)$$

from which we have

$$W_{c3}=l\pm\sqrt{l^2-2d}, \tag{2.27}$$

where

$$l=\frac{(\alpha+1)N_1+(1-\bar{\delta})(K+1)W_{п.ж1}}{2(\alpha+1)(K+1)}; \tag{2.28}$$

$$d=\frac{1}{2}\left[\frac{(1-\bar{\delta})N_1W_{п.ж1}}{(K+1)(\alpha+1)}+gb\right]; \tag{2.29}$$

$$N_1=Kn_{г1}+n_{п.ж1}+n_1; \tag{2.30}$$

$$b=R_cT_c. \tag{2.31}$$

The relationships of (2.27), (2.28), and (2.29) can also be obtained directly from expressions (2.18), (2.19), and (2.20) under the condition that $\kappa_c = 1$ and $i_{c0} = c_{p\ c}T_{c0}$.

2.4. Estimating Ejector Efficiency

The efficiency of gas-liquid ($p_{ж0н} < p_{г0н}$) and liquid-gas ($p_{ж0н} > p_{г0н}$) ejectors can be described by the efficiency, which is equal to the ratio of useful work to work spent:

$$\eta=\frac{L_{полезн}}{L_{затр}}. \tag{2.32}$$

Depending on what we understand as useful and spent work, different expressions are possible for determining ejector efficiency. Let us examine the most interesting of them.

1) <u>Adiabatic efficiency of a liquid-gas ejector</u> $\eta_{ад}^{ж.г}$. In this case the useful work is the work of adiabatic compression of the gas from initial total pressure $p_{г0н}$ to the total pressure of the mixture p_{c0}:

$$L_{полезн}=G_гJc_{p\ г}T_{г0н}\left[\left(\frac{p_{c0}}{p_{г0н}}\right)^{\frac{\varkappa_г-1}{\varkappa_г}}-1\right]; \tag{2.33}$$

work spent is the work of expanding the liquid from the initial total pressure $p_{ж0н}$ to the total pressure of the mixture p_{c0}:

$$L_{затр} = G_ж \frac{p_{ж0н} - p_{c0}}{\gamma_ж}. \tag{2.34}$$

If we substitute these expressions for $L_{полезн}$ and $L_{затр}$ in equation (2.32), then we get

$$\eta_{из}^{ж.г} = \frac{J\gamma_ж c_{p\,г} T_{г0н} K \left[\left(\frac{p_{c0}}{p_{г0н}}\right)^{\frac{\varkappa_г - 1}{\varkappa_г}} - 1\right]}{p_{ж0н} - p_{c0}}. \tag{2.35}$$

2) <u>Adiabatic efficiency of gas-liquid ejector</u> $\eta_{ад}^{г.ж}$. Useful and spent work in this case we determine by relationship

$$L_{полезн} = G_ж \frac{p_{c0} - p_{ж0н}}{\gamma_ж}; \tag{2.36}$$

$$L_{затр} = G_г J c_{p\,г} T_{г0н} \left[1 - \left(\frac{p_{c0}}{p_{г0н}}\right)^{\frac{\varkappa_г - 1}{\varkappa_г}}\right], \tag{2.37}$$

whose substitution in equation (2.32) gives us

$$\eta_{ад}^{г.ж} = \frac{p_{c0} - p_{ж0н}}{J c_{p\,г} \gamma_ж K T_{г0н} \left[1 - \left(\frac{p_{c0}}{p_{г0н}}\right)^{\frac{\varkappa_г - 1}{\varkappa_г}}\right]}. \tag{2.38}$$

3) <u>Isothermic efficiency of liquid-gas ejector</u> $\eta_{из}^{ж.г}$. In a number of cases (for example, in ejecting very hot gas by a large quantity of liquid) the most convenient results are provided by an estimate of the effectiveness of the liquid-gas ejector using the isothermic efficiency, which is equal to the ratio of useful work of isothermic compression of the gas from pressure $p_{г0н}$ to pressure p_{c0}

$$L_{полезн} = G_г R_г T_{c0} \ln \frac{p_{c0}}{p_{г0н}} \tag{2.39}$$

to the work spent in expanding the liquid from pressure $p_{ж0н}$ to pressure p_{c0}, which is determined according to formula (2.34). If we use expressions (2.39), (2.34), and (2.32), we find

$$\eta_{из}^{ж.г}=\frac{2{,}303\gamma_ж K R_г T_{c0}\log\frac{p_{c0}}{p_{г0н}}}{p_{ж0н}-p_{c0}}. \qquad (2.40)$$

4) <u>Isothermic efficiency of gas-liquid ejector</u> $\eta_{из}^{г.ж}$. Useful work in this case is determined by formula (2.36), and work spent, by relationship

$$L_{затр}=G_г R_г T_{c0}\ln\frac{p_{г0н}}{p_{c0}}. \qquad (2.41)$$

The isothermic efficiency of the gas-liquid ejector is equal to:

$$\eta_{из}^{г.ж}=\frac{p_{c0}-p_{ж0н}}{2{,}303\gamma_ж K R_г T_{c0}\log\frac{p_{г0н}}{p_{c0}}}. \qquad (2.42)$$

CHAPTER 3

CALCULATING FLOW IN THE NOZZLES

The equations obtained above enable us to find the parameters of a gas-liquid mixture at the outlet from the ejector provided that the parameters of the gas and the liquid in the inlet section of the mixing chamber are known. However, to calculate the characteristics of an ejector these equations are not enough. We must find additional conditions, which bind the parameters of the flows in the inlet and outlet sections of the nozzles and consider their interaction in the initial section of the mixing chamber. These conditions depend on the operational mode of the ejector and can vary substantially.

3.1. Calculating Parameters of a Gas Flow on the Nozzle Edge

3.1.1. Convergent Nozzle

The efficiency of the convergent nozzle is usually described by the velocity factor $\phi_{\Gamma 1H}$, which is equal to the ratio of gas velocities on the edge of the nozzle for real and ideal isentropic flows for the same ratio of static pressure $p_{\Gamma 1}$ on the nozzle edge to total pressure $p_{\Gamma 0H}$ at the inlet to the nozzle (see, for example, [1]):

$$\varphi_{\text{г}1\text{н}}=\frac{W_{\text{г}1}}{W_{\text{г}1\text{ид}}}=\frac{\lambda_{\text{г}1}}{\lambda_{\text{г}1\text{ид}}}. \tag{3.1}$$

For the nozzles which are generally used in ejectors we can assume $\phi_{\text{г}1\text{н}}$ = 0.98-0.99.

At a given ratio of pressures the quantities $\lambda_{\text{г}1}$ and $\lambda_{\text{г}1\text{ид}}$ can be found from

$$\left(1-\frac{\varkappa_{\text{г}}-1}{\varkappa_{\text{г}}+1}\lambda_{\text{г}1}^{2}\right)^{\frac{\varkappa_{\text{г}}}{\varkappa_{\text{г}}-1}}=p(\lambda_{\text{г}1})=\frac{p_{\text{г}1}}{p_{\text{г}01}}; \tag{3.2}$$

$$\left(1-\frac{\varkappa_{\text{г}}-1}{\varkappa_{\text{г}}+1}\lambda_{\text{г}1\text{ид}}^{2}\right)^{\frac{\varkappa_{\text{г}}}{\varkappa_{\text{г}}-1}}=p(\lambda_{\text{г}1\text{ид}})=\frac{p_{\text{г}1}}{p_{\text{г}0\text{н}}}, \tag{3.3}$$

while the coefficient of pressure recovery, which describes losses in the nozzle, is determined from formula

$$\nu_{\text{г}1\text{н}}=\frac{p_{\text{г}01}}{p_{\text{г}0\text{н}}}=\frac{p(\lambda_{\text{г}1\text{ид}})}{p(\lambda_{\text{г}1})}. \tag{3.4}$$

In a shut-off regime, when $\lambda_{\text{г}1}$ = 1, this formula has a conditional nature, since the geometry of real and ideal nozzles will be different. Actually, when $\lambda_{\text{г}1}$ = 1, as follows from expression (3.1), $\lambda_{\text{г}1\text{ид}}$ > 1, and thus the ideal nozzle must be divergent.

Total pressure losses in the convergent nozzle in a shut-off regime are described by quantity $\nu_{\text{г}1\text{н.р}}$, which is determined by

$$\nu_{\text{г}1\text{н.р}}=\left[\frac{\varkappa_{\text{г}}+1}{2}\left(1-\frac{\varkappa_{\text{г}}-1}{\varkappa_{\text{г}}+1}\frac{1}{\varphi_{\text{г}1\text{н}}^{2}}\right)\right]^{\frac{\varkappa_{\text{г}}}{\varkappa_{\text{г}}-1}}=\frac{p\left(\frac{1}{\varphi_{\text{г}1\text{н}}}\right)}{p_{\text{г}}(1)}, \tag{3.5}$$

where

$$p\left(\frac{1}{\varphi_{\text{г}1\text{н}}}\right)=\left(1-\frac{\varkappa_{\text{г}}-1}{\varkappa_{\text{г}}+1}\frac{1}{\varphi_{\text{г}1\text{н}}^{2}}\right)^{\frac{\varkappa_{\text{г}}}{\varkappa_{\text{г}}-1}}. \tag{3.6}$$

The flow of gas through the convergent nozzle is determined by formula (2.9), where $p_{\text{г}01}$ at an assigned value of $\phi_{\text{г}1\text{н}}$ is found by means of (3.4) or (3.5).

3.1.2. Divergent Nozzle

The divergent gas nozzle at an assigned relative area of the critical section $f_{\Gamma.\kappa} = f_{\Gamma.\kappa}/f_{\Gamma 1}$, depending on the ratio of static pressure p in the medium into which the stream flows to total pressure $p_{\Gamma 0 н}$ at the nozzle inlet, can operate in three substantially different regimes: 1) when the flow in the divergent part of the nozzle is supersonic when ($\lambda_{\Gamma 1} = \lambda_{\Gamma.p} > 1$); 2) when the flow in the divergent part of the nozzle is partially supersonic and partially subsonic and where the transition from the supersonic region of the flow, which borders the critical section of the nozzle, into the subsonic region, stretching to its outlet section, occurs in shocks ($\lambda_{\Gamma.\kappa} = 1$; $\lambda_{\Gamma 1} < 1$); 3) when the flow over the entire length of the nozzle is subsonic ($\lambda_{\Gamma.\kappa} < 1$; $\lambda_{\Gamma 1} < 1$).

The coefficient of pressure recovery in the supersonic nozzle can be represented in the form of the product of coefficients of pressure recovery in its convergent ($\nu_{\Gamma.\kappa.н}$) and divergent ($\nu_{\Gamma 1\kappa}$) parts:

$$\nu_{\Gamma 1 н}=\frac{p_{\Gamma 01}}{p_{\Gamma 0 н}}=\frac{p_{\Gamma 0 \kappa}}{p_{\Gamma 0 н}}\frac{p_{\Gamma 01}}{p_{\Gamma 0 \kappa}}=\nu_{\Gamma.\kappa.н}\nu_{\Gamma 1 \kappa}. \tag{3.7}$$

The efficiency of the convergent part of the nozzle can be described by $\phi_{\Gamma.\kappa.н} = \lambda_{\Gamma.\kappa}/\lambda_{\Gamma.\kappa.ид}$. Then $\nu_{\Gamma.\kappa.н}$ is determined by

$$\nu_{\Gamma.\kappa.н}=\frac{p(\lambda_{\Gamma.\kappa.ид})}{p(\lambda_{\Gamma.\kappa})}, \tag{3.8}$$

where reduced velocity $\lambda_{\Gamma.\kappa.ид}$ is found from formula

$$p(\lambda_{\Gamma.\kappa.ид})=\frac{p_{\Gamma.\kappa}}{p_{\Gamma 0 н}}=\left(1-\frac{\varkappa_\Gamma-1}{\varkappa_\Gamma+1}\lambda^2_{\Gamma.\kappa.ид}\right)^{\frac{\varkappa_\Gamma}{\varkappa_\Gamma-1}}. \tag{3.9}$$

In shut-off regimes in a supersonic nozzle, when $\lambda_{\Gamma.\kappa} = 1$, we have

$$\nu_{\Gamma.\kappa.н.p}=\left[\frac{\varkappa_\Gamma+1}{2}\left(1-\frac{\varkappa_\Gamma-1}{\varkappa_\Gamma+1}\frac{1}{\varphi^2_{\Gamma.\kappa.н}}\right)\right]^{\frac{\varkappa_\Gamma}{\varkappa_\Gamma-1}}=\frac{p\left(\frac{1}{\varphi_{\Gamma.\kappa.н}}\right)}{p_\Gamma(1)}. \tag{3.10}$$

Now let us introduce the relationships which determine the parameters of the flow on the edge of the divergent supersonic nozzle for all regimes mentioned above (see [7]).

Supersonic flow regime. This case is the most interesting, since it corresponds to the most advantageous operational regimes for a gas-liquid ejector.

The efficiency of the divergent part of the nozzle in this case can be described by the quantity

$$\varphi_{\text{г}1\text{к}}=\frac{\lambda_{\text{г.р}}-1}{\lambda_{\text{г.р.ид}}-1}; \qquad (3.11)$$

from which we get

$$\lambda_{\text{г.р}}=\varphi_{\text{г}1\text{к}}(\lambda_{\text{г.р.ид}}-1)+1, \quad (\lambda_{\text{г.р}}>1). \qquad (3.12)$$

The magnitude of the ideal reduced outflow velocity from the supersonic nozzle $\lambda_{\text{г.р.ид}} > 1$, which is contained in (3.11) and (3.12), is found from formula

$$q(\lambda_{\text{г.р.ид}})=q_{\text{г}}(1)\bar{f}_{\text{г.к}}. \qquad (3.13)$$

where $q(\lambda)$ is the gas dynamic function, which is determined by (1.40).

The coefficient of pressure recovery in the divergent part of the nozzle in the case of a supersonic gas flow can be found from relationship

$$\nu_{\text{г}1\text{кр}}=\frac{q(\lambda_{\text{г.р.ид}})}{q(\lambda_{\text{г.р}})}=\frac{q_{\text{г}}(1)\bar{f}_{\text{г.к}}}{q(\lambda_{\text{г.р}})}. \qquad (3.14)$$

If we consider (3.10) and (3.14), then from equation (3.7) for the studied case we get

$$\nu_{\Gamma 1\text{к.р}} = \frac{\varkappa_\Gamma + 1}{2} \frac{\tilde{f}_{\Gamma.\text{к}p}\left(\frac{1}{\varphi_{\Gamma.\text{к.н}}}\right)}{q(\lambda_{\Gamma.\text{р}})}, \tag{3.15}$$

where $\lambda_{\Gamma.\text{р}}$ is found with the aid of (3.12) and (3.13).

The velocity coefficient of the divergent part of the supersonic nozzle, which has rather smooth contours, can be assumed in the first approximation to be equal to 0.97-0.98.

In a one-dimensional study the calculation system for the flow in the supersonic nozzle is realized by changing pressure p in the surrounding space from zero to a certain limiting value

$$p_{\text{пр}} = p_{\Gamma 0\text{н}} \nu_{\Gamma 1\text{к.р}} \lambda_{\Gamma.\text{р}} q(\lambda_{\Gamma.\text{р}}) T\left(\frac{1}{\lambda_{\Gamma.\text{р}}}\right), \tag{3.16}$$

corresponding to the case where on the nozzle edge there develops a plane shock, in which the change in gas parameters is described by known relationships: $\lambda_{\Gamma 1}\lambda_{\Gamma 2} = 1$ and $p_{\Gamma 02}/p_{\Gamma 01} = q(\lambda_{\Gamma 1})/q(\lambda_{\Gamma 2})$.

Mixed flow regimes. These regimes develop at pressures in the surrounding medium which exceed the limiting pressure determined by (3.16). The coefficient of pressure recovery in the divergent nozzle in this case is uniquely determined by the magnitude of the reduced outflow velocity

$$\nu_{\Gamma 1\text{н}} = p\left(\frac{1}{\varphi_{\Gamma.\text{к.н}}}\right) \frac{\tilde{f}_{\Gamma.\text{к}}}{T_\Gamma(1)\, q(\lambda_{\Gamma 1})}, \tag{3.17}$$

where $\lambda_{\Gamma 1}$ can vary from $\lambda_{\Gamma 1\ max} = 1/\lambda_{\Gamma.\text{р}}$ to $\lambda_{\Gamma 1\ min}$, determined by the relationship

$$q(\lambda_{\Gamma 1\ min}) = q_\Gamma(1) \frac{\tilde{f}_{\Gamma.\text{к}}}{\nu_{\Gamma 1\text{к.д}}},\ (\lambda_{\Gamma 1\ min} < 1). \tag{3.18}$$

The mganitude of $\nu_{\Gamma 1\text{к.д}}$ contained in expression (3.18) is the coefficient of pressure recovery in the divergent part of the nozzle for a subsonic flow in the case of $\lambda_{\Gamma.\text{к}} = 1$.

The maximal value of the pressure in the surrounding medium, at which the studied flow system is still obtained, is determined by the relationship

$$p_{\max}=p_{\Gamma 1\max}=\frac{\varkappa_{\Gamma}+1}{2}f_{\Gamma.\kappa}p\left(\frac{1}{\varphi_{\Gamma.\kappa.\text{н}}}\right)p_{\Gamma 0\kappa}\frac{T(\lambda_{\Gamma 1\min})}{\lambda_{\Gamma 1\min}}. \qquad (3.19)$$

Subsonic flow regimes of gas in divergent nozzle. In this case the pressure recovery coefficient in the nozzle can be found from formula (3.7), where $\nu_{\Gamma.\kappa.\text{н}}$ at assigned values of $\phi_{\Gamma.\kappa.\text{н}}$ and $\lambda_{\Gamma.\kappa}$ is determined from (3.8), while the value $\nu_{\Gamma 1\kappa}$ is found from

$$p_{\Gamma 0\kappa}-p_{\Gamma 01}=\zeta_{1\kappa}\frac{\varrho_{\Gamma.\kappa}W^2_{\Gamma.\kappa}}{2}; \qquad (3.20)$$

from which we get

$$\nu_{\Gamma 1\kappa}=1-\frac{\varkappa_{\Gamma}}{\varkappa_{\Gamma}+1}\zeta_{1\kappa}\lambda^2_{\Gamma.\kappa}\varepsilon(\lambda_{\Gamma.\kappa}), \qquad (3.21)$$

where $\zeta_{1\kappa}$ is the resistance coefficient of the divergent part of the nozzle when $\lambda_{\Gamma.\kappa} < 1$, which is determined experimentally.

3.2. State Parameters and Speed of Sound in a Two-Phase Vapor-Liquid Mixture

When gas-liquid and liquid-gas ejectors work in limiting or near limiting regimes (see below) the static pressure of the liquid in the nozzle and in the initial part of the mixing chamber in some cases drops below saturation pressure $p_{s\ \text{н}}$ and the liquid begins to boil. In this case a two-phase vapor-liquid mixture is formed, which, depending on the concentration of vapor, can be either a mixture of liquid and fine vapor bubbles or a mixture of vapor and fine droplets, or, finally, a type of foam formation. In calculating the flow of such a mixture we will assume that it is also in a state of thermodynamic and mechanical equilibrium and, consequently, the velocities and temperatures

of the particles of vapor and liquid are the same at any point (the pressure of the vapor is assumed equal to the saturation pressure above the plane phase contact surface). The parameters of the components contained in the mixture (the boiling liquid and the dry saturated vapor) can be found under the assumption which we have taken by using the thermodynamic tables (see, for example, [9]).

The parameters of state of the studied vapor-liquid mixture, which is saturated vapor, are determined by the following relationships (see, for example, [6]):

$$v_{п.ж}=v'+x(v''-v'); \tag{3.22}$$

$$i_{п.ж}=i'+x(i''-i'); \tag{3.23}$$

$$s_{п.ж}=s'+x(s''-s'), \tag{3.24}$$

where v', i', s', and v'', i'', and s'' are the specific volumes, the enthalpies, and the entropies of the boiling liquid and dry saturated vapor, respectively; $x = G_п/(G_п + G_ж)$ is vapor content.

The speed of sound in the studied equilibrium vapor-liquid mixture can be determined from (1.23), which is reduced to the form of

$$a_{п.ж}=\sqrt{-gv^2_{п.ж}\left(\frac{dp}{dv_{п.ж}}\right)_{s=\text{const}}}. \tag{3.25}$$

If here we substitute the expression found in [10] for the derivative $\left(\frac{dp}{dv_{п.ж}}\right)_{s=\text{const}}$, then we get

$$a_{п.ж}=\frac{v_{п.ж}\sqrt{grJ}}{\sqrt{\begin{gathered}(v''-v')T\left\{\frac{v''-v'}{r}\left[T\frac{ds'}{dT}-x\left(\frac{dr}{dT}-\frac{r}{T}\right)\right]-\right.\\ \left.-x\frac{dv''}{dT}-(1-x)\frac{dv'}{dT}\right\}\end{gathered}}}, \tag{3.26}$$

where r is the latent heat of vapor formation.

The speed of sound in the boiling liquid during the transition from the moist vapor region ($x \to 0$) to the boiling liquid is equal to:

$$a_{п.ж(x\to 0)} = \frac{v' \sqrt{grJ}}{\sqrt{(v''-v')T\left[\frac{(v''-v')}{r}\frac{Tds'}{dT}-\frac{dv'}{dT}\right]}}. \quad (3.27)$$

For dry saturated vapor ($x \to 1$) we have, respectively,

$$a_{п.ж(x\to 1)} = \frac{v'' \sqrt{grJ}}{\sqrt{(v''-v')T\left\{\frac{(v''-v')}{r}\left[T\frac{ds'}{dT}+\frac{dr}{dT}-\frac{r}{T}\right]-\frac{dv''}{dT}\right\}}}. \quad (3.28)$$

Derivatives dv''/dT, dv'/dT, dr/dT, and ds'/dT, which are contained in these expressions, are approximately determined as the ratios of final increments in quantities $\Delta v''$, $\Delta v'$, Δr, $\Delta s'$ to the increment in temperature ΔT by means of the saturated-vapor tables. Since quantity p and T for saturated vapor are uniquely bound to each other, then the speed of sound $a_{п.ж}$ is a function of only two independent parameters, for example, T and x.

The dependences $a_{п.ж}(t)$ when $x = const$ for a moist water vapor, calculated from formula (3.26) by means of tables [9] when $\Delta t = 1°$, are shown in Fig. 3. If we compare Fig. 3 to Fig. 1 we see that the dependences of $a_{п.ж}(x)$ when $p = const$ and $t = const$ differ substantially from the dependences of the speed of sound a_c in the gas-liquid mixture by the relative concentration of gas in it.

As the gas content decreases with $p = const$ and $t = const$, quantity a_c first decreases to a certain minimal value, then rises, and when $K \to 0$ it tends to infinity, at the same time that quantity $a_{п.ж}$ decreases monotonically with a decrease in the vapor content, and when $x \to 0$ it reaches a certain minimal value, which depends on t or p.

During the transition from the region of moist saturated vapor to the liquid region the speed of sound rises intermittently from $a_{п.ж} \to 0$, which is determined by (3.27), to a value corresponding to the speed of sound in the liquid.

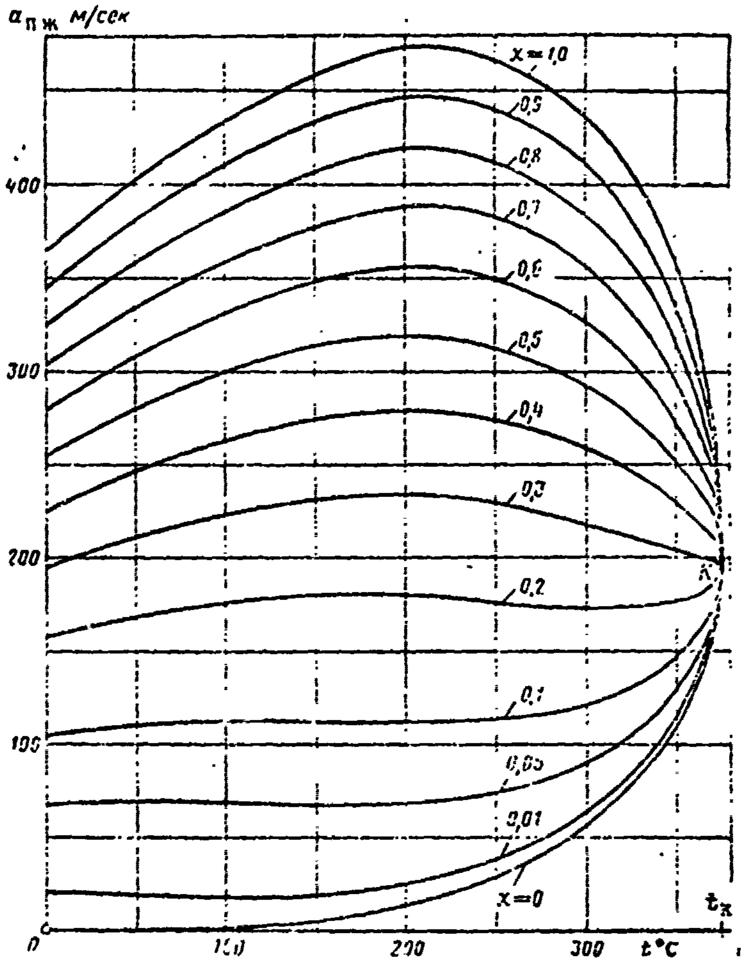


Fig. 3. Dependences of speed of sound in moist saturated water vapor on temperature when x = const.
Desigantion: м/сек = m/s.

It should be mentioned that because of the presence of gases dissolved in the liquid, which might be liberated in the case of a pressure decrease, and also in connection with the fact that in a rapid pressure change (which occurs, for example, during the flow of a spontaneously evaporating liquid in a nozzle or diffuser) the processes of evaporation and condensation will be delayed and the actual values of the speed of sound in the saturated vapor, particularly in the transition region between vapor and liquid ($x \to 0$) and the reverse transition, might differ noticeably from the values found from formulas (3.26) and (3.27).

3.3. Calculating Liquid Flow in a Nozzle

3.3.1. Convergent Nozzle

As the liquid flows from the convergent nozzle two cases are possible: 1) when pressure p_1 at the edge of the nozzle exceeds saturation pressure ($p_1 > p_{s\,н}$) and 2) when $p_1 < p_{s\,н}$.

Case 1. In this case the liquid in the nozzle does not evaporate. If its compressibility can be ignored, then the outflow velocity from the nozzle is determined according to formula

$$W_{ж1}=\varphi_{ж1н}W_{ж1ид}=\varphi_{ж1н}\sqrt{2gv_{ж}(p_{ж0}-p_{ж1})}, \qquad (3.29)$$

where $\varphi_{ж1н}=W_{ж1}/W_{ж1ид}$ is the velocity coefficient.

Total pressure and temperature on the nozzle edge can be found from

$$p_{ж01}=p_{ж0н}\left[\frac{p_{ж1}}{p_{ж0н}}(1-\varphi^2_{ж1н})+\varphi^2_{ж1н}\right]; \qquad (3.30a)$$

$$T_{ж1}=T_{ж.н}+\Delta T_{ж}\approx T_{ж.н}+\frac{1}{Jc_{ж}\gamma_{ж}}(p_{ж0н}-p_{ж01}). \qquad (3.30b)$$

If the temperature of the liquid is close to its value at the critical point, then condition $v_{ж}$ = const becomes roughly approximate. At the assigned parameters of state of the liquid at the nozzle inlet and static pressure at its edge, the outflow velocity is found with the aid of the i-s diagram from formula

$$W_{ж1}=\varphi_{ж1н}W_{ж1ид}=\varphi_{ж1н}\sqrt{2gJ(i_{ж0н}-i_{ж1ид})}. \qquad (3.31)$$

Then we determine the enthalpy of the liquid on the nozzle edge:

$$i_{ж1}=(1-\varphi^2_{ж1н})i_{ж0н}+\varphi^2_{ж1н}i_{ж1ид}, \qquad (3.32)$$

the specific weight $\gamma_{ж1}$, and entropy $s_{ж1}$. Total pressure $p_{ж01}$ on the nozzle edge can be found with the tables according to the values of $i_{ж01} = i_{ж0н}$ and $s_{ж01} = s_{ж1}$.

Case 2. In this case in the outlet section of the nozzle - between section s, where the pressure of the liquid becomes equal to saturation pressure ($p_{ж\ s} = p_s$), and the outlet section of the nozzle 1 - spontaneous evaporation of the liquid occurs. The outflow velocity of the vapor-liquid mixture is determined by relationship

$$W_{п.ж1} = \varphi_{ж1н} W_{п.ж1ид} = \varphi_{ж1н} \sqrt{2gJ(i_{ж0н} - i_{п.ж1ид})} = \sqrt{2gJ(i_{ж0н} - i_{п.ж1})}, \qquad (3.33)$$

from which we get

$$i_{п.ж1} = (1 - \varphi^2_{ж1н}) i_{ж0н} + \varphi^2_{ж1н} i_{п.ж1ид}. \qquad (3.34)$$

The value of $i_{п.ж1ид}$ which is contained in these expressions at assigned values for the state parameters of the liquid at the nozzle inlet and static pressure in its outlet section is found by the aid of the saturated-vapor tables from condition $s_{п.ж1ид} = s_{ж0н}$, which gives us

$$i_{п.ж1ид} = i'_1 + x_{1ид}(i''_1 - i'_1) = i'_1 + \frac{s_{ж0н} - s'_1}{s''_1 - s'_1}(i''_1 - i'_1). \qquad (3.35)$$

The vapor content of the mixture is determined from

$$x_1 = \frac{1}{i''_1 - i'_1}\left[(1 - \varphi^2_{ж1н}) i_{ж0н} + \varphi^2_{ж1н} i_{п.ж1ид} - i'_1\right], \qquad (3.36)$$

after which, according to formulas (3.22) and (3.24) the quantities $v_{п.ж1}$ and $s_{п.ж1}$ are calculated.

In cases where the compressibility of the liquid in a range of variation from $p_{ж0н}$ to $p_{s\ н}$ can be ignored, expression (3.33) is reduced to a more convenient form for calculation:

$$W_{п.ж1}=\varphi_{ж1н}\sqrt{2gJ(i_{ж.н\,s}-i_{п.ж1ид})+W_s^2}, \qquad (3.37)$$

where the magnitude of W_s is determined according to formula

$$W_s=\varphi_{ж1н}\sqrt{2gv_ж(p_{ж0н}-p_{ж.н\,s})}. \qquad (3.38)$$

Quantities $i_{ж.н\,s}$ and $p_{ж.н\,s}$, contained in these expressions, represent the enthalpy and pressure, respectively, of the boiling liquid in nozzle section s flowing without losses ($T_{ж\,s} = T_{ж.н}$).

As the pressure decreases in the medium into which the stream of liquid is ejected from the value $p = p_{ж0н}$ to a certain minimal value, the outflow rate and the flow rate of the liquid rise steadily and their values can be found from the condition $p = p_{ж1} = p_{н.ж1}$. Finally, when $p = p_{min}$ the convergent nozzle becomes cut off: a further decrease in pressure in the surrounding medium from p_{min} to zero no longer results in an increased flow rate of liquid or a change in the parameters of the flow on the edge of the nozzle. Depending on the total pressure of the liquid $p_{ж0н}$ at the inlet, when $T_{ж.н}$ = const three shut-off regimes are possible for the liquid nozzle: 1) when $p_1 \geq p_{s1}$ the outflow rate is equal to the speed of sound in the liquid ($W_{ж1} = a_{ж1}$); 2) when $p_1 = p_{s0}$ the velocity of the liquid on the edge of the nozzle is greater than the speed of sound $a_{п.ж1(x\to0)}$; 3) when $p_1 < p_{s\,н}$ the outflow velocity of the vapor-liquid mixture is equal to the speed of sound in it ($W_{п.ж1} = a_{п.ж1}$).

In the first case of shut-off the fluid flows along the entire convergent nozzle. Since the speed of sound in the liquid is very great, then this case is realized at extremely high pressures of the liquid at the nozzle inlet. For example, the speed of sound in water under normal atmospheric conditions is equal to $a_ж$ = 1445 m/s. If we substitute this value of the velocity of the liquid in the equation of energy conservation, we find

$$p_{ж0н}=10\,000+\frac{1000\cdot 1445^2}{2g}=1{,}065\cdot 10^8 \text{ kgf/m}^2=10650 \text{ kgf/cm}^2.$$

The maximal flow rate of the liquid is found in this case from the relationship

$$G_{max}=\gamma_{ж1}a_{ж1}f_{ж1}.$$

The studied shut-off case is realized at values of $p_{ж0н}$ which exceed $p_{ж0I}$, at which the conditions $W_{ж1} = a_{ж1}$ and $p_{ж1} = p_{s1}$ are simultaneously fulfilled.

In the second shut-off case the liquid also flows over the entire length of the nozzle. Its velocity, if we ignore the temperature change in the liquid due to friction, can be found from formula (3.38) or from relationship (3.31) when $i_{ж1ид} = i_{s\ н} = i'_{н}$. The maximal flow rate of the liquid is determined in this case by relationship $G_{max} = \gamma_{ж\ s}W_{ж\ s}f_{ж1}$. At the assigned temperature of the liquid $T_{ж.н}$ in front of the nozzle this shut-off case is possible at values of $p_{ж0н}$ which range from $p_{ж0I}$ to $p_{ж0II}$, at which the outflow velocity of the liquid from the nozzle $W_{ж\ s1}$ becomes equal to the limiting value of the speed of sound in the vapor-liquid mixture when $x \to 0$. The value of $p_{ж0II}$ when $v_ж$ = const depends only on the physical properties in the temperature of the liquid and can be found from the obvious relationship

$$p_{ж0II}=\frac{\gamma'_1 a^2_{п.ж1(x\to 0)}}{2g}+p_{s1}. \qquad (3.39)$$

where the speed of sound $a_{п.ж(x\to 0)}$ is found from formula (3.27).

It is interesting to note that when $p_{ж0I} > p_{ж0н} > p_{ж0II}$ the pressure in the stream of liquid on the edge of the nozzle [illegible] decreased counterpressure cannot drop lower than saturation pressure p_s by a finite value, since the flow on the edge of the nozzle becomes supersonic ($W_{ж\ s} > a_{п.ж1(x\to 0)}$) as the slightest

amount of vapor is developed and perturbations can no longer penetrate into the nozzle.

In the third shut-off case, when $p_{ж1} < p_{s\ н}$, a stream of moist vapor is ejected from the nozzle. Evidently this case can only develop in the region in front of the nozzle where the total pressure of the liquid changes from $p_{ж0II}$ to $p_{ж0III} = p_{s1} \approx p_{s\ н}$. The state parameters of the moist vapor and the outflow velocity in the shut-off regime can be found from condition $W_{п.ж1} = a_{п.ж1}$.

The calculation is performed in the following sequence.

1) A series of values is assigned for pressure p_1 ranging from $p_{s\ н}$ to zero.

2) From the saturated-vapor tables we find the values corresponding to them T, v', v", i', i", s', s", and r, after which from formulas (3.33), (3.34), (3.35), (3.36), (3.22) and (3.24) we determine quantities $W_{п.ж1}$, $i_{п.ж1}$, x_1, $v_{п.ж1}$ and $s_{п.ж1}$.

3) From formula (3.26) we find the value of the speed of sound $a_{п.ж1}$, after which we graphically determine quantities $p_{п.ж1}$ and $W_{п.ж1}$, which correspond to the shut-off regime.

4) For this value of $p_{п.ж1}$ we calculate all the parameters of the mixture on the nozzle edge in the same sequence.

Figure 4 shows dependences $p_{ж0II}(t)$ and $p_{ж0III}(t) = p_s(t)$ for water. We see that the third shut-off regime of the convergent nozzle, which corresponds to a flow of liquid with spontaneous evaporation, is only possible at total pressures in the liquid which are very close to the saturation pressure. Thus, for example, when $t_{ж.н}$ = 100, 200, and 300°C we have, respectively: $p_{ж0II}$ = 1.0394 kgf/cm^2, p_s = 1.0332 kgf/cm^2; $p_{ж0II}$ = 16.540 kgf/cm^2, p_s = 15.857 kgf/cm^2 and $p_{ж0II}$ = 99.15 kgf/cm^2, p_s = 87.61 kgf/cm^2.

The main shut-off regime of the convergent nozzles obtained in the ejector will therefore be the second regime, in which $p_{ж1} = p_{s1}$.

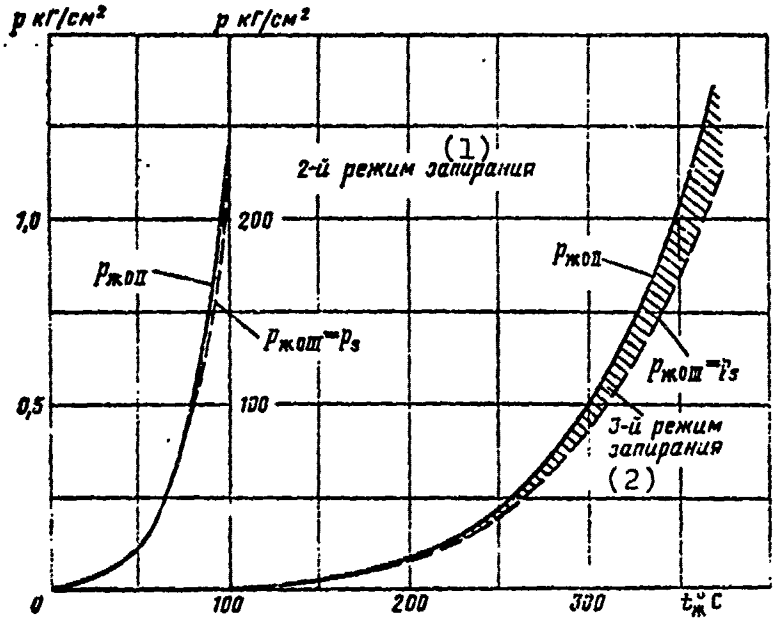


Fig. 4. Dependences of total pressure of the liquid on temperature for limiting shut-off regimes of the nozzle.
KEY: (1) 2nd shut-off regime; (2) 3rd shut-off regime.
Designation: кГ/см^2 = kgf/cm^2.

3.3.2. Divergent Nozzle

As the liquid flows in a divergent nozzle with an assigned relative critical section area $\bar{f}_{ж.к} = f_{ж.к}/f_{ж1}$, just as in the gas flow, three different regimes are possible:

1) the rated regime, in which a supersonic flow of moist saturated vapor is obtained in the divergent part of the nozzle;

2) a regime in which regions of supersonic and subsonic flows, separated by a plane shock, develop in the divergent part of the nozzle;

3) a subsonic flow regime for the entire length of the nozzle.

Supersonic flow regime. When $T_{ж.н}$ = const there are three possible cases of supersonic flow in the divergent nozzle (Fig. 5), depending on the total pressure value of the liquid $p_{ж0н}$.

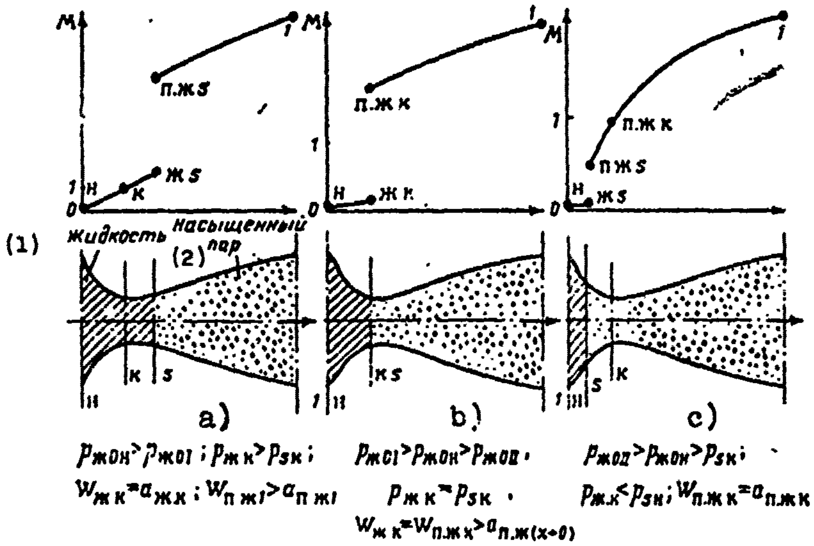


Fig. 5. Three cases of supersonic flow in divergent nozzle:

$a - p_{ж0н} > p_{ж0I}, \; p_{ж.к} > p_{sк};$
$b - p_{ж0I} > p_{ж0н} > p_{ж0II};$
$c - p_{ж0II} > p_{ж0н} > p_{sк}$

KEY: (1) Saturated vapor; (2) Liquid.

We have the first case when $p_{ж0н} > p_{ж0I}$ (see Fig. 5a). In this case the liquid flows in the convergent part of the nozzle and its velocity in the critical section $W_{ж.к}$ is equal to the speed of sound in the liquid $a_{ж.к}$ ($M_{ж.к} = 1$). In the initial section of the divergent part of the nozzle (between sections к–s)

the pressure of the liquid decreases to p_s, while its velocity increases and becomes supersonic ($M_{ж\ s} > 1$). In section s evaporation of the liquid begins and the speed of sound decreases intermittently from $a_{ж\ s}$ to $a_{п.ж(x\to 0)}$. For this reason the M number of the flow rises sharply ($M_{п.ж\ s} >> M_{ж\ s}$). In section s-1 of the nozzle the velocity of the supersonic flow of the two-phase vapor-liquid mixture and number $M_{п.ж}$ rise smoothly, while pressure falls.

In the second case (see Fig. 5b), which develops when $p_{ж0I} > p_{ж0н} > p_{ж0II}$, the static pressure of the liquid in the critical section of the nozzle is equal to saturation pressure ($p_{ж.н} = p_{s\ н}$), where $W_{ж.н} < a_{ж.н}$ ($M_{ж.н} < 1$). In the critical section of the nozzle the liquid begins to evaporate, and the speed of sound in the two-phase mixture which has formed $a_{п.ж(x\to 0)}$ is lower than the velocity of the mixture $W_{п.ж.н} = W_{ж.н}$, and thus the M number increases intermittently and the flow becomes supersonic ($M_{п.ж.н} > 1$). In the divergent part of the nozzle the velocity and the M number of the supersonic flow which has formed from the two-phase mixture rise steadily ($M_{п.ж1} > M_{п.ж.н}$). Thus, in the studied case the transition from the subsonic region of the flow to the supersonic region in the critical section of a divergent nozzle occurs intermittently. The velocity of the flow is not equal to the speed of sound at a single one of the points along the nozzle.

In the third case (see Fig. 5c), which is possible at values of $p_{ж0н}$ ranging between $p_{ж0II}$ and $p_{s\ н}$, between sections н-s of the convergent part of the nozzle the static pressure of the liquid decreases to the value p_s, while the velocity increases to the value W_s. In section s the liquid begins to evaporate, and thus, as in the cases described above, the M number of the flow rises intermittently, although the flow remains subsonic ($M_{п.ж\ s} < 1$). Between sections s-н the velocity of the formed two-phase mixture increases and in the critical section becomes equal to the speed of sound ($M_{п.ж.н} = 1$). In the divergent part of the nozzle quantity $M_{п.ж}$ rises steadily.

In the convergent part of a supersonic nozzle the flow is calculated just as in the case described above for the flow in the convergent nozzle in a shut-off regime. A supersonic flow in the divergent part of the nozzle with assigned geometry and with friction losses can be calculated if we use the equations of the conservation of mass and energy, which are written in the form of

$$W_{п.ж1р}=\frac{G_{max}}{f_{ж1}}v_{п.ж1}; \tag{3.40}$$

$$\left(\frac{G_{max}}{f_{ж1}}\right)^2 v^2_{п.ж1}+2gJi_{п.ж1}=2gJi_{ж0н}, \tag{3.41}$$

as follows:

1) If we assign a series of values $p_{п.ж1} < p_{п.ж.к}$ and determine with the aid of the saturated-vapor tables the values corresponding to them

$$x_1=\frac{s_{н.ж.к}-s_1'}{s_1''-s_1'} \tag{3.42}$$

and values $v_{п.ж1}$ and $i_{п.ж1}$ from formulas (3.22) and (3.23), then with the aid of equations (3.41) and (3.40) we find the parameters of state $p_{п.ж1ид}$, $i_{п.ж1ид}$, $v_{п.ж1ид}$ and the velocity of the mixture $W_{п.ж1ид}$ on the edge of the nozzle for a flow without losses in its divergent part.

2) According to formula

$$W_{н.ж1р}=\varphi_{ж1к}(W_{п.ж1ид}-W_{н.ж.к})+W_{н.ж.к} \tag{3.43}$$

we calculate the outflow velocity from the nozzle in the presence of losses, after which we find quantities $v_{п.ж1}$ and $i_{п.ж1}$ from expressions (3.40) and (3.41).

3) If we assign a series of values for $p_{п.ж1}$, which somewhat exceed the value $p_{п.ж1ид}$, then using equation

$$\frac{i_{п.ж1} - i_1'}{i_1'' - i_1'} = \frac{v_{п.ж1} - v_1'}{v_1'' - v_1'} \qquad (3.44)$$

and the saturated-vapor tables we find the pressure on the edge of the nozzle, and then from relationship (3.22) and (3.24) - quantities x_1 and $s_{п.ж1}$.

Mixed flow regimes. In these regimes in the divergent part of the nozzle a plane shock develops, which transforms the supersonic saturated vapor flow into a subsonic flow of liquid or saturated vapor, which depends on the location of the shock, determined by the magnitude of counterpressure.

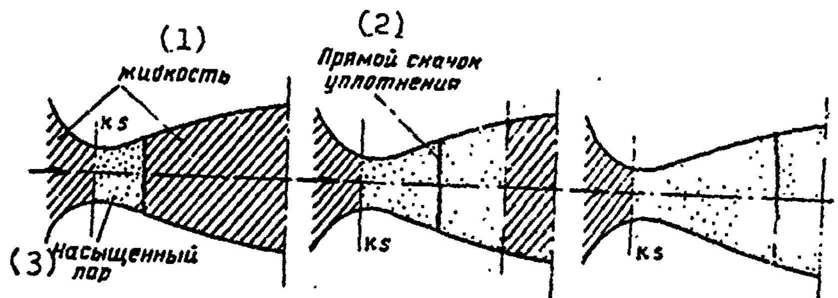


Fig. 6. Possible mixed flow systems in divergent nozzle when $p_{ж0I} > p_{ж0н} > p_{ж0II}$.
KEY: (1) Liquid; (2) Plane shock; (3) Saturated vapor.

Figure 6 shows the possible flow systems in the nozzle for these regimes in the case where $p_{ж0I} > p_{ж0н} > p_{ж0II}$.

Figure 7 gives the T-s part of the diagram, in which the dashed line indicates the process of expansion of the liquid in the nozzle from the initial state, which is described by quantities $p_{ж0н}$, $i_{ж0н}$, and $T_{ж.н}$ (point н), to states corresponding to the points 1a, 1b, 1c, 1d and 1e, at which a plane shock develops in the nozzle. The values of the state parameters beyond the plane

shock lie along the dot-dash line (points 2a, 2b, 2c, 2d, and 2e). Points 3a, 3b, 3c, 3d, and 3e, lying along the line $i_{жОн}$ = const, correspond to the stagnation parameter of the flow beyond the plane shock.

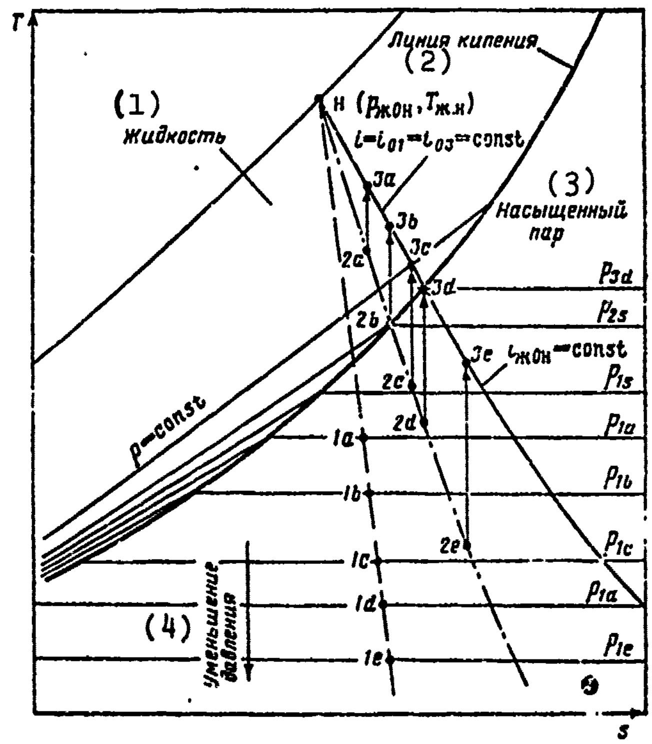


Fig. 7. Processes of liquid expansion in nozzle with formation of a supersonic flow of moist vapor and stagnation of the supersonic flow in a plane shock.
KEY: (1) Liquid; (2) Boiling line; (3) Saturated vapor; (4) Pressure decrease.

If we examine Fig. 7, we see that if the plane shock is located near the critical section total condensation of the vapor occurs within it (points 1a, 2a, 3a; see also Fig. 6a). As the shock moves from the critical section and as pressure decreases in front of it, the pressure of the liquid behind the shock, beginning at a certain moment, also decreases and, finally, becomes equal to the saturation pressure (points 1b, 2b, 3b in Fig. 7). As the shock moves further in the direction of the outlet section of the nozzle and with a further decrease in pressure in front of

it, the pressure behind the shock drops below the saturation pressure and none of the vapor is condensed; as the flow of saturated vapor stagnates in the divergent part of the nozzle, pressure in this case can rise above the saturation pressure and the vapor can be completely condensed (points 1c, 2c, 3c, 1d, 2d, 3d; also see Fig. 6b). At even lower pressures ahead of the shock a subsonic flow of saturated vapor develops behind it, which is not condensed even when the stagnation velocity decreases to zero (points 1e, 2e, 3e; also see Fig. 6c).

Let us assign the calculation order for the parameters of state in the velocity of the flow on the edge of the divergent nozzle for the studied regimes depending on the pressure on the edge, which is equal to pressure p in the surrounding medium.

1. Let us find the parameters of state and the velocity of the flow in the critical and outlet sections of the nozzle in the supersonic flow regime.

2. Let us calculate the parameters of state and the velocities of the flow behind a plane shock, located in the outlet section of the nozzle (see following section 3.4).

If total vapor condensation occurs in the shock, then over the entire possible range of variation in counterpressure, which corresponds to the studied flow system, liquid will flow from the nozzle. If for the sake of simplicity we assume that $\gamma_{ж}$ = const, then we get

$$W_{ж1}=\frac{G_{max}}{\gamma_{ж1}f_{ж1}}, \qquad (3.45)$$

from which it follows that the outflow rate is independent of counterpressure; stagnation pressure $p_{ж01}$ is determined by the Bernoulli equation:

$$p_{ж01}=p_{ж1}+\frac{\gamma_{ж}W_{ж1}^{2}}{2g}. \qquad (3.46)$$

Quantity $p_{ж1}$ in mixed flow regimes in the studied case can vary from a minimal value of $p_{ж1\ min}$, which corresponds to a plane shock located in the outlet section of the nozzle (see Section 2), to a maximal value of $p_{ж1\ max}$, which corresponds to a case where the plane shock moves into the critical section and where the liquid flows over the entire length of the divergent part of the nozzle.

Quantity $p_{ж1\ max}$ is determined from formula

$$p_{ж1 max} = p_{ж0к} - \frac{\gamma_ж}{2g}\left(\zeta_{ж1к} W^2_{зк} - W^2_{ж1}\right), \tag{3.47}$$

where $\zeta_{ж1к}$ is the resistance coefficient of the divergent part of the nozzle for the liquid flow.[1]

In a case where the vapor remains entirely uncondensed behind the plane shock in the outlet section of the nozzle (points 2c, 2d, 2e in Fig. 7), the flow on the edge of the nozzle under different counterpressures is calculated in the following manner.

A number of values are given for pressure p ranging from the maximal, which is found from relationship (3.47), to the minimal, which is equal to pressure behind the plane shock on the nozzle edge (see Section 2). When pressure p changes from $p_{ж1\ max}$ to $p_{ж1} = p_{s1}$ a stream of liquid flows from the nozzle, whose velocity and total pressure when $\gamma_ж$ = const are determined from formulas (3.45) and (3.46); its temperature is determined from (3.30b). Quantity p_{s1} is found as the intersection point of curves $T_{ж1}(p_{ж1})$ and $T_s(p_s)$. When pressure changes from p_{s1} to $p_{ж1\ min}$ saturated vapor flows from the nozzle. Its specific

[1]In the case of $p_{ж0II} > p_{ж0н} > p_{s\ н}$ in the critical section and in the small region behind it a vapor flow is possible, although in determining the quantity $p_{ж1\ max}$ we can use formula (3.47) in the first approximation.

volume and enthalpy when $p_{п.ж1}$ = const are determined from the combined solution of equations (3.41) and (3.44), its outflow velocity - from (3.40).

Subsonic flow regimes. These regimes are realized at pressure values on the nozzle edge which exceed the value $p_{ж1\ max}$, found according to formula (3.47). The parameters of these regimes can be calculated by means of the relationships presented above.

3.4. Plane Shock Inflow of Saturated Vapor

Flow parameters behind a plane shock, which transform a supersonic saturated vapor flow into a subsonic saturated vapor flow or into a liquid are found from the combined solution of the equations of the conservations of mass, momentum, and energy for sections in front of and behind the shock, written in the form of

$$v_2 = \frac{W_2}{gm}; \tag{3.48}$$

$$p_2 = n - mW_2; \tag{3.49}$$

$$i_2 = i_{ж0н} - \frac{W_2^2}{2gJ}. \tag{3.50}$$

Quantities $m = \rho_{п.ж1} W_{п.ж1}$ and $n = mW_{п.ж1} + p_{п.ж1}$, which are contained in these equations, depend only on the parameters of the impinging flow; the subscript "2" denotes flow parameters behind the shock, subscript "1" - parameters in front of the shock.

Since we do not know in advance whether or not a liquid flow or a saturated vapor flow will form behind the shock, the calculation is made in two stages.

First we assume that a liquid flows behind the shock. If we assign a number of values $v_{ж2}$ ranging from $v_{ж01}$ to $v'_{ж1}$, then from expressions (3.48), (3.49), and (3.50), we find the corresponding values $W_{ж2}$, $p_{ж2}$, and $i_{ж2I}$. Using the thermodyanmic tables from values $p_{ж2}$ and $v_{ж2}$ we find the values of $i_{ж2II}$ which correspond to them. Comparison of these values with the values of $i_{ж2I}$ which we have found enables us to determine the sought values $p_{ж2}$, $i_{ж2}$, $v_{ж2}$, $W_{ж2}$, and $s_{ж2}$. Then quantity $p_{ж02}$ can be found from the tables from the known values of the enthalpy and entropy of stagnation $i_{ж02} = i_{ж0н}$ and $s_{ж02} = s_{ж2}$.

If over the entire range of possible change in quantity $v_{ж2}$ dependences $i_{ж2I}(v_{ж2})$ and $i_{ж2II}(v_{ж2})$ do not intersect, then a liquid flow behind the shock is impossible.

Calculation of the flow parameters behind a plane shock is simplified if the liquid is considered incompressible. In this case velocity $W_{ж2}$ and pressure $p_{ж2}$ behind the shock are uni... determined from equations (3.48) and (3.49), while the stagnation temperature $p_{ж02}$ is found from the Bernoulli equation:

$$p_{ж02} = p_{ж2} + \frac{\gamma_ж W^2_{ж2}}{2g} \tag{3.51}$$

The temperature of the liquid behind the shock can be found according to formula

$$T_{ж2} = T_{ж01} + \frac{p_{ж01} - p_{ж02}}{Jc_{pж}\gamma_ж}. \tag{3.52}$$

It is obvious that the flow of fluid behind the shock can only develop in a case where quantity $T_{ж2}$ is lower than saturation temperature T_s when $p_s = p_{ж2}$.

In the case of a flow of saturated vapor behind the shock, the parameters behind the shock will be calculated in the following sequence:

1) if we assign a series of values $p_{п.ж2}$ ranging from $p_{п.ж1}$ to p' with $i = i_{ж01}$, then from expressions (3.49), (3.48), and (3.50) we find values $W_{п.ж2}$, $v_{п.ж2}$, $i_{п.ж2I}$, which correspond to them;

2) if we compare this value of $i_{п.ж2I}$, with the value of $i_{п.ж2II}$, which is determined from

$$i_{п.ж2II} = i_2' + (i_2'' - i_2') \cdot \frac{v_{п.ж2} - v_2'}{v'' - v'}, \tag{3.53}$$

then we find quantities $p_{п.ж2}$ and $i_{п.ж2}$, after which we calculate velocity $W_{п.ж2}$, specific volume $v_{п.ж2}$, as well as entropy $s_{п.ж2}$ and vapor content x_2 of the mixture behind the shock by formulas (3.49), (3.48), (3.23) and (3.24).

CHAPTER 4

CALCULATING CHARACTERISTICS FOR EJECTOR WITH DIVERGENT DIFFUSER

4.1. Possible Operational Modes of Ejector

Let us describe the possible working modes of the ejector. With this object we examine its choking characteristics, which represent the dependence of total pressure of the mixture on the ejection coefficient under invariable parameters of the gas and the liquid at the ejector inlet. Typical choking characteristics of a two-phase ejector for different assignments of gas and liquid parameters at the inlet are shown in Fig. 8.

The gas-liquid mixture is a compressible medium, and thus under assigned state parameters for the gas and the liquid at the ejector inlet it is always possible, if we decrease counterpressure, to obtain a regime in which the flow in the divergent diffuser between section 3-4 (see Fig. 2) will be completely supersonic. Point 1 on the curves corresponds to this regime.

As counterpressure in the supersonic stream flowing from the diffuser increases, increasingly powerful angle shock develops and, finally, the perturbations penetrate inside the channel. In a one-dimensional study penetration of the disturbance inside

the channel is only possible from the moment that the plane shock develops in the outlet section (point 2 of curves in Fig. 8). It is obvious that the total-pressure values of the mixture in the outlet section of the diffuser in regimes corresponding to points 1 and 2 on the curves are distinguished from one another by a value which is close to the losses in the plane shock.

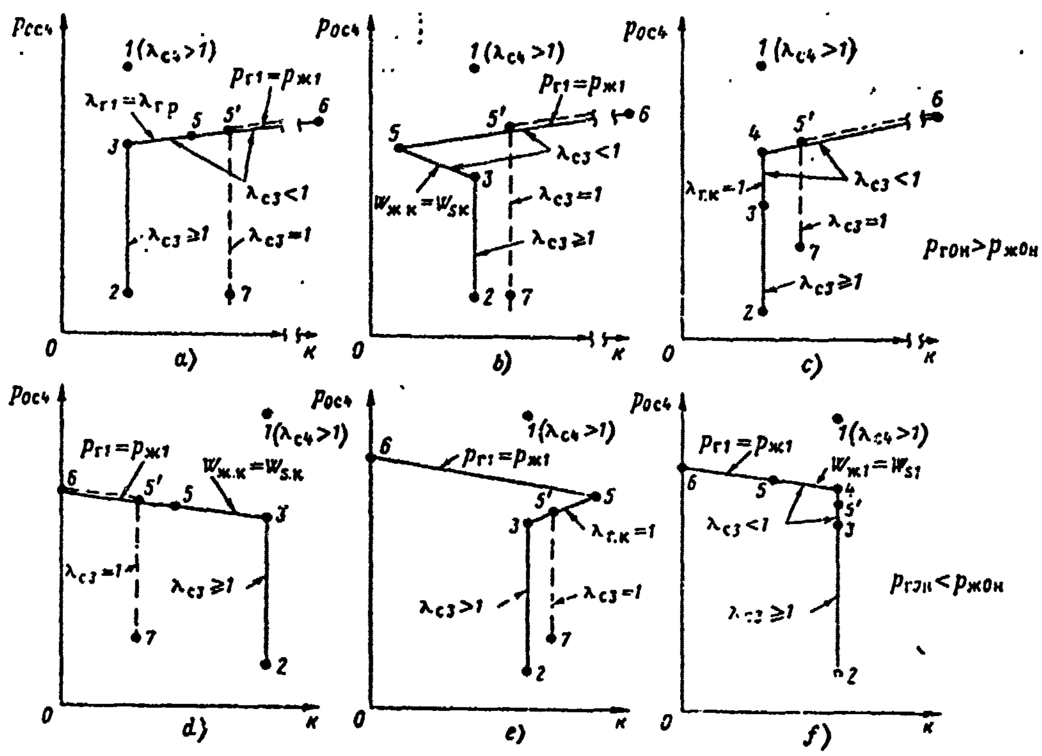


Fig. 8. Choking characteristics of two-phase ejector for different assignments of gas and liquid parameters at inlet.

As counterpressure increases from values corresponding to points 2 of the curves the normal shock moves deep within the diffuser and, finally, into the outlet section of the mixing chamber. Here total pressure of the mixture increases steadily, since losses in the plane shock decrease. Sections 2-3 of the curves correspond to these modes.

Due to the fact that with a change in the total pressure of the mixture from the value corresponding to point 1 to values corresponding to points 2 and 3, the flow in the mixing chamber of the ejector does not change, and the ejection coefficient also remains invariable. Thus, point 1 and section 2-3 of the curve lie along a single vertical line.

If we ignore the contrast in losses in the diffuser for supersonic and subsonic flows, then in the case of a low relative volume of liquid in the mixture, the total pressure value at point 3 of the curve can be expressed as the value which corresponds to pressure at point 1, as follows:

$$p_{c04III} = p_{c04I} \frac{q(\lambda'_{c3})}{q\left(\frac{1}{\lambda'_{c3}}\right)}, \qquad (4.1)$$

where $\lambda'_{c3} > 1$.

In the particular case, when the velocity of the mixture in the outlet section of the mixing chamber is equal to the speed of sound ($\lambda_{c3} = 1$) points 1 and 3 coincide.

Regimes corresponding to the points of the vertical branches of the curves, at which the flow rates of the gas and the liquid do not depend on the conditions at the diffuser outlet, are called the cut-off [choking] regimes of the ejector.

In gas-liquid and liquid-gas ejectors with convergent nozzles, whose trailing edges have a zero thickness ($\bar{\delta} = 0$), two types of cut-off regimes can be realized, depending on the state parameters of the gas and the liquid:

1) critical regimes in which the flow at the outlet from the mixing chamber is supersonic ($\lambda_{c3} > 1$);

2) cut-off regimes of the mixing chamber in which the velocity at the outlet from the chamber is equal to the critical velocity ($\lambda_{c3} = 1$).

The limiting critical regime and the limiting cut-off regime of the mixing chamber are also those regimes in which the flow in the diffuser is completely subsonic. In a case where the edges of the convergent nozzle have a zero thickness the limiting critical regime and the limiting cut-off regime of the mixing chamber correspond to the vertices of the vertical branches of the curve (points 3 in Fig. 8a, b, d, e).

In an ejector with a superconic gas nozzle and a convergent liquid nozzle, in addition to the critical regimes and cut-off regimes of the mixing chamber, in a certain variation range of the parameters of state of the gas and the liquid cut-off regimes in the nozzles can also develop in which the velocity of the mixture in the outlet section of the mixing chamber and in the diffuser will be subsonic (the region of the subsonic flow which does not occupy the entire cross section of the mixing chamber extends up to the edge in the cut-off modes of the nozzles). In these regimes a sonic stream of moist vapor flows from the convergent nozzle. This stream expands suddenly in the initial section of the mixing chamber and becomes supersonic. In the divergent part of the gas nozzle a normal shock develops, which leads to a supersonic flow in the subsonic part. Depending on counterpressure the position of the shock can change, and thus when K = const quantities λ_{r1}, p_{r01}, and, consequently, p_{c04} can change. Sections 3-4 of the choking characteristics correspond to the cut-off modes of the nozzles in Fig. 8c and f.

The cut-off modes of the nozzles can also develop in an ejector with a convergent gas nozzle when its edges have a finite thickness. In this case the change in value p_{c04} when K = const and $\lambda_{c3} < 1$ occurs due to the fact that with an increase in counterpressure as compared to the value corresponding to the

limiting critical regime, there is an increase in pressure p_1 in the stagnant regions, formed at the trailing edges of the nozzle.

It should be mentioned that the transition from the supersonic region of the flow to the subsonic in a diffuser does not actually occur in a plane shock, but in complex systems of bridge shocks, in which losses, generally speaking, can be distinguished from losses in a plane shock. However, as tests on gas and liquid-gas vacuum ejectors with cylindrical mixing chambers indicate, when the chamber is sufficiently long (on the order of 12-18 calibre) losses in these shock systems in the limiting critical regimes are very close to losses in the plane shock.

With an increase in counterpressure as compared to counterpressure values corresponding to points 3 (see Fig. 8a, b, d, e) or 4 (see Fig. 8c and f) from the diffuser the disturbances penetrate into the convergent parts of the nozzles, and, depending on the nature of the flow in the initial part of the mixing chamber, there begins a decrease in either the flow rate of the liquid (see Fig. 8a, c, e) or in the gas (see Fig. 8b, d, f) or in both simultaneously (see Fig. 8 - dashed curves). The total pressure of the mixture in this case rises monotonically, while the ejection coefficient either rises steadily (see Fig. 8a and c), decreases steadily (see Fig. 8d and f), or first decreases (rises), and then rises (decreases) (see Fig. 8b and e). In the regime $K = \infty$ the flow rate of the liquid is equal to zero and gas alone flows through the ejector. When $K = 0$ only liquid flows through the ejector ($G_г = 0$).

The curve shown in Fig. 8a, b, c corresponds to the gas-liquid ejector ($p_{г0н} > p_{ж0н}$), while the curves in Fig. 8d, e, f correspond to the liquid-gas ejector ($p_{ж0н} > p_{г0н}$).

Regimes in which the change in counterpressure is accompanied by a change in the ejection coefficients are called subcritical regimes. The sloping segments of the choking curves 6-3 or 6-4 in Fig. 8 correspond to these regimes. Obviously when the ejector works in subcritical modes all the way from the inlet to the outlet section, there will be a continuous subsonic flow region through which perturbations will be transmitted.

4.2. Critical Regimes

The critical operational regimes of two-phase gas-liquid and liquid-gas ejectors, as we have already mentioned, are those regimes in which the flow in the outlet section of the mixing chamber is supersonic. In studying the critical regimes of a two-phase ejector we assume that when one of the streams on the edge of the nozzle is sonic or supersonic and the other is subsonic, then the latter, as a result of its compression by the diverging supersonic stream, is driven in the initial part of the mixing chamber (between sections 1-2, see Fig. 2) to the maximal possible velocity.

In a case where the stream of liquid is subsonic, then when $p_{ж0I} > p_{ж0н} > p_{ж0II}$ its velocity increases to the value W_{s2}, and when $p_{ж0II} > p_{ж0н} > p_{s2}$, when during compression of the liquid stream it is evaporated, the velocity of the stream increases to a value equal to the speed of sound $\alpha_{п,ж2}$ in the stream of saturated vapor.

In a case where the stream of saturated vapor or gas is subsonic, then its velocity in the initial section of the mixing chamber rises to the speed of sound (this case is obtained in critical regimes of a supersonic gas ejector - see, for example, [7]). Section 2 (see Fig. 2) of the mixing chamber, in which the velocity of the subsonic stream reaches the maximal possible value, is called the cut-off section.

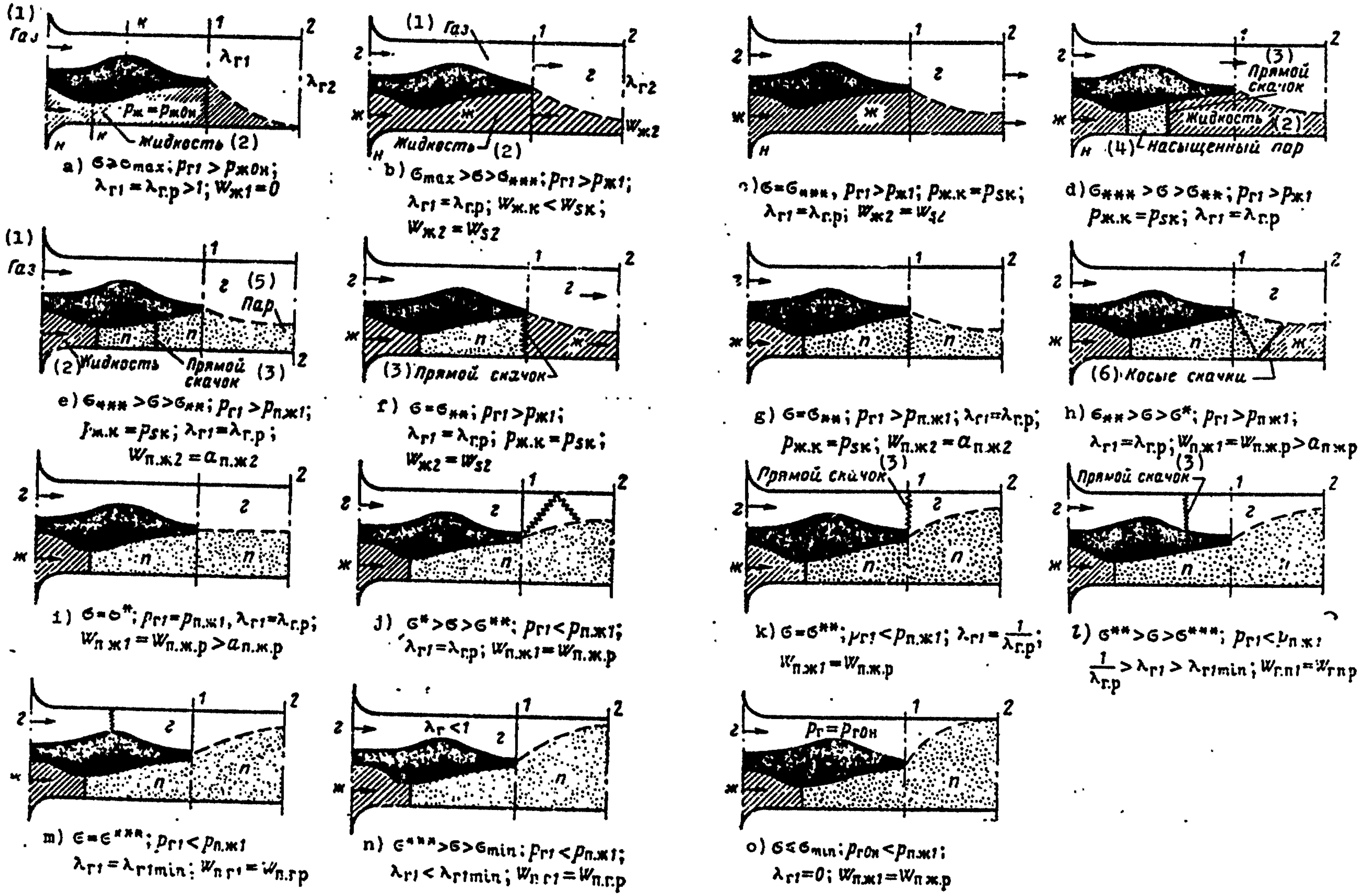


Fig. 9. Theoretically possible flow patterns in nozzles and in the initial section of the mixing chamber for ejector working in critical regimes.
KEY: (1) Gas; (2) Liquid; (3) Plane shock; (4) Saturated vapor; (5) Vapor; (6) Angle shocks.

Figure 9 shows theoretically possible systems of a flow in nozzles and in the initial section of the mixing chamber, corresponding to the critical operational modes of a two-phase ejector with supersonic nozzles at different values of the characteristic pressure ratio $\sigma = p_{\text{г0н}}/p_{\text{ж0н}}$ and invariable values $T_{\text{г0н}}$, $p_{\text{ж0н}}$ and $T_{\text{ж.н}}$. The patterns of flow in the liquid nozzle in cut-off regimes and regimes close to it correspond in Fig. 9 to the case where $p_{\text{ж0I}} > p_{\text{ж0н}} > p_{\text{ж0II}}$. The patterns of flow in the initial section of the mixing chamber and the conditions of stream interaction remain the same for cases where $p_{\text{ж0н}} > p_{\text{ж0I}}$ and $p_{\text{ж0II}} > p_{\text{ж0н}} > p_{s2}$ (see Figs. 5 and 6).

At values of characteristic pressure ratio σ which exceed a certain maximal value σ_{max} the static pressure of the gas on the edge of the nozzle in a rated flow system, when $\lambda_{\text{г}1} = \lambda_{\text{г.р}} > 1$, greatly exceeds the pressure of the liquid, and thus the supersonic gas stream, as it expands in the initial section of the mixing chamber, fills its entire cross section (see Fig. 9a). The flow rate of the liquid in this case is equal to zero ($G_{\text{ж}} = 0$; $K = \infty$). The ejection process when $\sigma \geq \sigma_{max}$ is impossible.

When $\sigma_{max} > \sigma > \sigma_{***}$ (see Fig. 9b) the static pressure of the gas at the edge of the nozzle for the rated flow pattern exceeds, just as before, the pressure of the liquid ($p_{\text{г}1} > p_{\text{ж}1}$), although the supersonic gas flow, as it expands in the initial section of the mixing chamber, no longer fills its cross section. The pressure of the liquid in the critical section of the nozzle in these regimes exceeds the saturation pressure, and thus liquid flows for the entire length. The velocity of the liquid in the initial section of the mixing chamber increases and, in keeping with the basic hypothesis of critical regimes, reaches the maximal possible value for a convergent nozzle $W_{\text{ж}2} = W_{s2}$ in Section 2.

With a decrease in σ from σ_{max} to σ_{***} the divergent region of the gas flow in the initial section of the mixing chamber decreases, and thus the area of the cross section of the liquid

stream $f_{ж2}$ and the flow rate of the liquid increase. In this case the velocity of the liquid in the critical section also increases. When $\sigma = \sigma_{***}$ the liquid nozzle is cut-off: the static pressure of the liquid in the critical section becomes equal to the saturation pressure ($p_{ж.к} = p_{s\ к}$), while the velocity and the flow rate of the liquid reach maximal values ($W_{ж.к} = W_{s\ к}$, $G_ж = G_{ж\ max}$) (see Fig. 9c). With a further decrease in σ the flow in the convergent part of the liquid nozzle remains unchanged ($p_{ж.к} = p_{s\ к}$, $W_{ж.к} = W_{s\ к}$, $G_ж = G_{ж\ max}$).

In a σ variation range from σ_{***} to σ_{**} the flow scheme shown in Fig. 9d and e is obtained. In the initial section of the mixing chamber, just as in the preceding cases, the supersonic gas flow ($\lambda_{г1} = \lambda_{г.р}$; $p_{г1} > p_{ж1}$) expands and the flow from the liquid nozzle contracts. In connection with the continuing increase in the area $f_{ж2}$ and decrease in pressure on the edge of the liquid nozzle, a supersonic flow of saturated vapor develops in its divergent part, which is transformed into a subsonic flow in the plane shock, which when $\sigma = \sigma_{**}$ reaches the outlet section of the nozzle (see Fig. 9f and g). At values which are not too low for the relative area of the critical section of the liquid nozzle $\tilde{f}_{ж.к} = f_{ж.к}/f_{ж1}$ in the plane shock there occurs a total condensation of vapor, regardless of the position of the shock in the nozzle, i.e., for the entire σ variation range from σ_{***} to σ_{**}. Here, in the initial section of the mixing chamber a liquid stream flows, whose velocity in section 2 is equal to W_{s2} (see Fig. 9d and f). At values of $f_{ж.к}$ which are lower than a certain minimal value, a flow of liquid on the edge of the nozzle and between sections 1-2 of the mixing chamber is only possible with a change in σ from σ_{***} to a certain value of σ_{***1}, at which the pressure of the liquid on the edge of the nozzle becomes equal to the saturation pressure. Here the flow schemes shown in Fig. 6a and b develop in the liquid nozzle. At values of σ between σ_{***1} and σ_{**} a subsonic stream of saturated vapor flows from the

nozzle. The velocity of this stream in the initial section of the mixing chamber increases to the speed of sound ($W_{п.ж2} = a_{п.ж2}$, see Fig. 9e and g).

When $\sigma_{**} > \sigma > \sigma^*$ (see Fig. 9h) supersonic streams flow from both nozzles ($\lambda_{г1} = \lambda_{г.р}$; $W_{п.ж1} > a_{п.ж1}$), and in the initial section of the mixing chamber the gas stream expands and the stream of saturated vapor contracts ($p_{г1} > p_{п.ж1}$). As σ decreases $p_{г1}$ tends to $p_{п.ж1}$ and when $\sigma = \sigma^*$ pressures on the edge of the nozzles become identical ($p_{г1} = p_{п.ж1}$); the streams which flow from the nozzles when $\sigma = \sigma^*$ have a cylindrical shape (see Fig. 9i).

With a further decrease in σ the flow pattern in the gas and liquid nozzles and in the streams in the initial part of the mixing chamber undergoes a reverse change. From the liquid nozzle flows a supersonic stream of saturated vapor, whose static pressure $p_{п.ж1}$ exceeds the pressure in the gas stream $p_{г1}$, and thus it expands and constri-ts the gas flow.

When $\sigma^* > \sigma > \sigma^{**}$ a supersonic stream, in which angle shocks develop (see Fig. 9j), flows from the gas nozzle. With a decrease in σ the intensity of the angle shocks increases, and when $\sigma = \sigma^{**}$ on the edge of the gas nozzle a plane shock is established (see Fig. 9k). The subsonic gas stream which develops behind the plane shock is driven between sections 1-2 from $\lambda_{г1} = 1/\lambda_{г.р}$ to the speed of sound ($\lambda_{г2} = 1$).

At even lower values of σ the plane shock penetrates into the divergent part of the gas nozzle and, finally, when $\sigma = \sigma^{***}$ it reaches its critical section. The flow pattern for regimes with $\sigma^{**} > \sigma > \sigma^{***}$ and $\sigma = \sigma^{***}$ are shown in Fig. 9l and m, respectively.

When $\sigma^{***} > \sigma > \sigma_{min}$ over the entire length of the gas nozzle, the flow is subsonic ($\lambda_{г.к} < 1$) With a decrease in σ the range of expansion of the supersonic stream of vapor increases and,

finally, when $\sigma = \sigma_{min}$ the stream of vapor fills the entire cross section of the mixing chamber. When $\sigma \leq \sigma_{min}$ the flow rate of the gas is equal to zero ($G_{г} = 0$; $K = 0$) and the ejection process is impossible.

Let us find the conditions which bind the flow parameters on the edge of the gas and liquid nozzles for an ejector working in critical modes for all of the flow schemes above. With this goal let us examine the flow in the initial part of the mixing chamber between sections 1-2 (see Fig. 9) for cases where one of the streams at the inlet to the mixing chamber is subsonic and the other is sonic or supersonic. In addition to the assumptions made in deriving the ejection equations and in studying the flow of gas and liquid in the nozzles, let us assume that:

1) the streams in the initial part of the mixing chamber do not mix;

2) there is no heat exchange between the streams;

3) the flow is isentropic;

4) both streams in the cut-off section are one-dimensional, and the velocity vectors are parallel to the ejector axis.

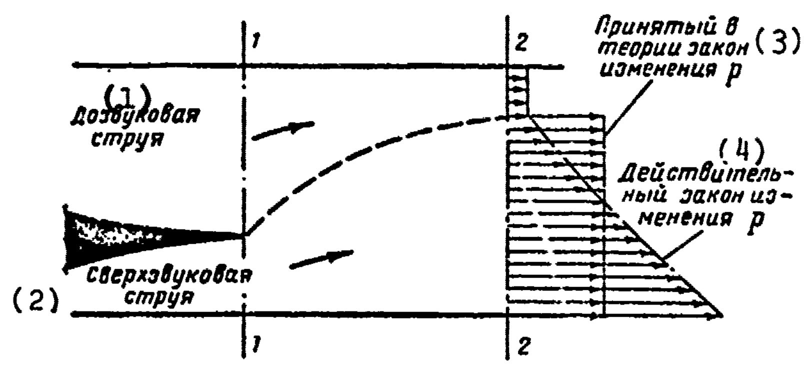


Fig. 10. Replacing the actual pattern of distribution of static pressure in the cut-off section by a step-wise pattern - with a break on the flow boundary.
KEY: (1) Subsonic stream; (2) Supersonic stream; (3) Pattern of change in p assumed in theory; (4) Actual pattern of change in p.

According to this last assumption, the known pattern of static pressure distribution in the cut-off section and, consequently, the actual pattern of velocity distribution, is replaced, just as in the theory of a gas ejector (see [ ]) by a step-wise pattern with a break on the flow boundary (Fig. 10).

Let us solve jointly the equations for the conservation of mass, momentum, and energy for sections 1-2.

According to our assumption $p_{r02} = p_{r01}$ and $T_{r02} = T_{r01}$. Considering this, from the equation of the conservation of mass for the gas stream we get

$$q(\lambda_{r2})\bar{f}_{r2}=q(\lambda_{r1})\bar{f}_{r1}. \tag{4.2}$$

The analogous relationship for the vapor-liquid stream is written as

$$\gamma_{п.ж2}W_{п.ж2}\bar{f}_{п.ж2}=\gamma_{п.ж1}W_{п.ж1}\bar{f}_{ж1}. \tag{4.3}$$

If we use relationships (2.3) and (2.4), as well as the obvious condition

$$\bar{f}_{r2}+\bar{f}_{п.ж2}=1, \tag{4.4}$$

then from (4.2) and (4.3) we find

$$1-\frac{\alpha(1-\bar{\delta})}{\alpha+1}\frac{q(\lambda_{r1})}{q(\lambda_{r2})}=\frac{1-\bar{\delta}}{\alpha+1}\frac{\gamma_{п.ж1}W_{п.ж1}}{\gamma_{п.ж2}W_{п.ж2}}. \tag{4.5}$$

The equation of momentum for sections 1-2 can be represented in the following form:

$$G_{r1}\left(W_{r1}+\frac{gp_{r1}f_{r1}}{G_{r1}}\right)+G_{п.ж1}\left(W_{п.ж1}+\frac{gp_{п.ж1}f_{ж1}}{G_{п.ж1}}\right)+\delta g p_{t}=$$
$$=G_{r2}\left(W_{r2}+\frac{gp_{r2}f_{r2}}{G_{r2}}\right)+G_{п.ж2}\left(W_{п.ж2}+\frac{gp_{п.ж2}f_{п.ж2}}{G_{п.ж2}}\right). \tag{4.6}$$

If we consider that $G_{г2} = G_{г1} = G_{г.н}$; $G_{п.ж2} = G_{п.ж1} = G_{ж.н}$ and $T_{г02} = T_{г0н}$, then we transform this equation by means of expression (2.12) into the form of

$$\frac{\varkappa_г + 1}{2\varkappa_г} Ka_{к.г.н}[z(\lambda_{г1}) - z(\lambda_2)] = n_{п.ж2} - n_{п.ж1} - n_1, \tag{4.7}$$

where

$$n_{п.ж2} = W_{п.ж2} + \frac{gp_{п.ж2}v_{п.ж2}}{W_{п.ж2}}, \tag{4.8}$$

while the sets $n_{п.ж1}$ and n_1 are determined from formulas (2.13) and (2.14).

The velocity of the saturated vapor in the cut-off section, which is contained in equations (4.5) and (4.7), is found from the equation of the conservation of energy

$$\frac{W^2_{п.ж2}}{2gJ} + i_{п.ж2} = \frac{W^2_{п.ж1}}{2gJ} + i_{п.ж1} = i_{ж0н}, \tag{4.9}$$

which gives us

$$W_{п.ж2} = \sqrt{2gJ(i_{ж0н} - i_{п.ж2})}. \tag{4.10}$$

At the assigned value of $p_{п.ж2}$ of the saturated vapor in the cut-off section quantities $v_{п.ж2}$ and $i_{п.ж2}$ can be found from expressions (3.22) and (3.23), and the value of the vapor content, contained in these expressions, can be found from condition $s_{п.ж2} = s_{п.ж1}$ from formula

$$x_2 = \frac{s_{п.ж1} - s'_2}{s''_2 - s'_2}, \tag{4.11}$$

Equations (4.5), (4.7) and (4.9) can be used in calculating the critical regime in the variation ranges of σ from σ_{max} to σ_{**} and from σ^{**} to σ_{min}.

In the σ variation range from σ_{max} to σ_{**} at the assigned temperature of the liquid, depending on pressure $p_{ж0н}$ and quantity $f_{ж.к}$ in the initial section of the mixing chamber, three flow schemes are possible for the stream flowing from the liquid nozzle:

1) when a liquid flows between sections 1-2 (see Fig. 9b, c, d, f);

2) when a liquid flows from the nozzle, but when partial evaporation occurs between sections s-2;

3) when there is a subsonic flow of saturated vapor between sections 1-2 (see Fig. 9e, g).

The first case is realized at total-pressure values of the liquid $p_{ж01}$ which exceed quantity $p_{ж01II}$, which is determined from formula (3.39), where subscript "1" corresponds to the parameters of the liquid on the edge of the nozzle. In this case additional conditions which determine the flow of the streams in the initial part of the mixing chamber will be: $\lambda_{г1} = \lambda_{г.р}$; $x_1 = x_2 = 0$; $p_{ж2} = p_{s2}$ $(W_{ж2} = W_{s2})$.

In the second case, which arises at total-pressure values $p_{ж01}$ between $p_{ж01II}$ and p_{s2}, the additional conditions will be: $\lambda_{г1} = \lambda_{г.р}$; $x_1 = 0 (p_{ж1} > p_{s1})$; $W_{п.ж2} = a_{п.ж2}$.

In the third case the additional conditions will be the same as in the second, the only difference being that $x_1 > 0$.

In the σ variation range from σ^{**} to σ_{min}, when in the initial part of the mixing chamber the supersonic stream of saturated vapor expands and the subsonic stream of gas contracts, the additional conditions for the system of critical regime equations will be: $W_{п.ж1} = W_{п.ж.р} > a_{п.ж1}$ and $\lambda_{г1} = 1$.

At the assigned values for the geometrical parameters of the ejector, as well as for the physical properties and stagnation parameters of the gas and the liquid at the nozzle inlet, in order to calculate the critical regime we must know in which of the above ranges the assigned value of the characteristic pressure relationship is found. For this purpose we must compare the assigned value of σ with the values σ_{max}, σ_{***}, σ_{**}, σ^{*}, σ^{**}, σ^{***} and σ_{min}. In determining these values $p_{\text{г}0\text{н}}$ can be considered variable. Let us assign the calculation order for these quantities.

The quantity σ_{max} is found from equations (4.7) and (4.5) with the aid of expressions (2.10), (2.13), (2.14) and (4.8), assuming that $\lambda_{\text{г}1} = \lambda_{\text{г.р}}$ and that at the limit, as the flow rate of the liquid tends to zero, $W_{\text{ж}1} \to 0$, $p_{\text{ж}1} \to p_{\text{ж}0\text{н}}$:

$$\sigma_{max} = \left[1 + \frac{\overline{\delta}(\alpha+1)}{1-\overline{\delta}} \frac{p_1}{p_{\text{ж}0\text{н}}}\right] \frac{1}{\alpha \nu_{\text{г}1\text{н.р}} q(\lambda_{\text{г.р}}) [z(\lambda_{\text{г}2\,max}) - z(\lambda_{\text{г.р}})]}, \quad (4.12)$$

where the value of $\lambda_{\text{г}2\ max}$ is found from formula

$$q(\lambda_{\text{г}2\,max}) = \frac{\alpha(1-\overline{\delta})}{\alpha+1} q(\lambda_{\text{г.р}}). \quad (4.13)$$

Quantities σ_{***} and σ_{**} can be found from equations (4.7) and (4.5), which, if conditions $\lambda_{\text{г}1} = \lambda_{\text{г.р}}$ and $\nu_{\text{г}1\text{н}} = \nu_{\text{г}1\text{н.р}}$ are considered, is written in the form

$$\sigma = \frac{p_{\text{г}0\text{н}}}{p_{\text{ж}0\text{н}}} = \frac{\gamma_{\text{п.ж}1} W_{\text{п.ж}1} (n_{\text{п.ж}1} + n_1 - n_{\text{п.ж}2})}{g \alpha p_{\text{ж}0\text{н}} \nu_{\text{г}1\text{н.р}} q(\lambda_{\text{г.р}}) [z(\lambda_{\text{г}2}) - z(\lambda_{\text{г.р}})]}; \quad (4.14)$$

$$q(\lambda_{\text{г}2}) = \frac{\alpha(1-\overline{\delta}) q(\lambda_{\text{г.р}}) \gamma_{\text{п.ж}2} W_{\text{п.ж}2}}{(\alpha+1) \gamma_{\text{п.ж}2} W_{\text{п.ж}2} - (1-\overline{\delta}) \gamma_{\text{г.ж}1} W_{\text{г.ж}1}}. \quad (4.15)$$

Unknown quantities $\gamma_{\text{п.ж}1}$, $W_{\text{п.ж}1}$, $\gamma_{\text{п.ж}2}$, $W_{\text{п.ж}2}$, $n_{\text{п.ж}1}$, n_1 and $n_{\text{п.ж}2}$, contained in these formulas, can be found by calculating the flow of liquid in the nozzle and between sections 1-2 of the mixing chamber.

When $\sigma = \sigma_{***}$ these quantities can be calculated, in a case where the compressibility of the liquid can be ignored, in the following sequence:

1) we determine the maximal flow rate of the liquid through the nozzle G_{max} and quantity $p_{ж0к}$ (see Section 3.3), after which, from formulas (3.45), (3.47), (3.46), (3.30b), (2.13) and (2.14), we calculate quantities $W_{ж1}$, $p_{ж1}$, $p_{ж01}$, $T_{ж1}$, $n_{ж1}$, n_1 and from the saturated-vapor tables we find the saturation pressure $p_{s1} = p_{s2}$;

2) from expressions (3.27) and (3.39) we find quantities $a_{п.ж(x \to 0)}$ and $p_{ж01II}$;

3) if $p_{ж01} \geq p_{ж01II}$, then $x_2 = 0$, $p_{ж2} = p_{s1}$, $\gamma_{ж2} = \gamma_2'$ and the velocity $W_{п.ж2}$ is calculated according to formula

$$W_{п.ж2} = W'_{s2} = \sqrt{2gv_2'(p_{ж01} - p_{s1})}, \tag{4.15a}$$

after which from (4.8) we determine quantity $n_{п.ж2} = n_{s2}$;

4) if $p_{ж01II} > p_{ж01} > p_{s1}$, then in the cut-off section flows a sonic stream of saturated vapor. Quantities $p_{п.ж2}$, $\gamma_{п.ж2}$ and $W_{п.ж2} = a_{п.ж2}$ are determined in the same order as the third cut-off case of the liquid nozzle (see Section 3.3).

When $\sigma = \sigma_{**}$ the parameters of the stream which flows from the liquid nozzle in sections 1 and 2 is calculated as follows:

1) we determine the state parameters and the velocity of the stream of saturated vapor on the edge of the supersonic liquid nozzle in its rated operational regime (see Section 3.3), after which, with the aid of relationships presented in Section 3.4, we find the parameters of the subsonic flow of liquid or saturated vapor behind the shock located in the outlet section of the nozzle, as well as quantities $n_{п.ж1}$ and n_1;

2) if in the shock the vapor is completely condensed, then we calculate the parameters of the liquid stream in section 2 according to parts 2, 3, 4 of the preceding calculation;

3) if a subsonic flow of saturated vapor develops behind the shock, then the state parameters of the vapor in section 2 are calculated from conditions $s_{п.ж2} = s_{п.ж1}$ and $W_{п.ж2} = a_{п.ж2}$ according to part 4 of the preceding calculation.

When $\sigma = \sigma^*$ supersonic streams ($W_{п.ж1} = W_{п.ж.р} > a_{п.ж1}$; $\lambda_{г1} = \lambda_{г.р} > 1$) with identical pressures flow from both nozzles. From the condition of equality of pressures $p_{г1} = p_{п.ж1}$ we have

$$\sigma^* = \frac{p_{п.ж.р}}{p_{ж0н}} \frac{1}{\nu_{г1н.р} p(\lambda_{г.р})}, \tag{4.16}$$

where $p_{п.ж.р}$ is the static pressure on the edge of the liquid nozzle for a supersonic outflow velocity.

Quantities σ^{**} and σ^{***} can be found from equations (4.7) and (4.5), which are written as

$$\sigma = \frac{\gamma_{п.ж.р} W_{п.ж.р} (W_{п.ж.р} + n_1 - n_{п.ж2})}{g \alpha p_{ж0н} \nu_{г1н} [\text{[illegible]}\, z(1) - z(\lambda_{г1})]}; \tag{4.17}$$

$$\gamma_{п.ж.} W_{п.ж2} = \frac{(1-\bar{\delta}) q_г(1) \gamma_{п.ж.р} W_{п.ж.р}}{(\alpha+1) q_г(1) - \alpha(1-\bar{\delta}) q(\lambda_{г1})}, \tag{4.18}$$

where the subscript "p" denotes the flow parameters of the saturated vapor on the edge of the nozzle in a supersonic outflow.

When $\sigma = \sigma^{**}$ the additional conditions will be:

$$\lambda_{г1} = 1/\lambda_{г.р} \text{ and } \nu_{г1н} = \nu_{г1н.р} q(\lambda_{г.р})/q(1/\lambda_{г.р}),$$

and when $\sigma = \sigma^{***}$ - relationships (3.17), (3.18).

Quantities σ^{**} and σ^{***} according to formulas (4.17) and (4.18) are calculated, after determining the parameters of state and the

velocity of the supersonic stream of saturated vapor on the edge of the nozzle, as follows.

A series of values $p_{п.ж2}$ are assigned ranging from $p_{п.ж.р}$ to zero and values v_2', v_2'', i_2', i_2'', s_2', s_2'', which correspond to them, are found from the saturated vapor tables. According to formulas (4.11), (3.22), (3.23) and (4.10) we calculate the corresponding values of x_2, $\gamma_{п.ж2} = 1/v_{п.ж2}$, $i_{п.ж2}$ and $W_{п.ж2}$. By determining the values of the product $\gamma_{п.ж2}W_{п.ж2}$ from equation (4.18) we find the sought value $p_{п.ж2}$, after which w calculate the quantities $\gamma_{п.ж2}$, $W_{п.ж2}$, $n_{п.ж2}$ corresponding to it and, finally, values σ^{**} and σ^{***} according to formula (4.17).

When $\sigma \to \sigma_{min}$ we have: $\lambda_{г1} \to 0$; $\nu_{г1н} \to 1$. From equations (4.17) and (4.18) in this case we get

$$\sigma_{min} = \frac{\gamma_{п.ж.р} W_{п.ж.р} (n_{п.ж.р} + n_1 - n_{п.ж2})}{g a p_{ж0н}}; \tag{4.19}$$

$$\gamma_{п.ж2} W_{п.ж2} = \frac{(1-\overline{\tau})\, q_г(1)\, \gamma_{п.ж.р} W_{п.ж.р}}{(a+1)\, q_г(1)}. \tag{4.20}$$

We calculate quantity σ_{min} just as quantities σ^{**} and σ^{***}.

After determining the range in which lie the assigned value of σ and, consequently, the additional conditions which determine the flow of the streams in the nozzle and in the initial part of the mixing chamber, we calculate the parameters of the stream on the edge of the nozzles in the critical operational mode as follows.

Case $\sigma_{max} > \sigma > \sigma_{**}$

1. We determine the parameters of the supersonic gas flow on the nozzle edge.

2. We assign a series of values for static pressure on the edge of the liquid nozzle in the necessary range and determine the corresponding parameters of state and velocity of the liquid or vapor-liquid nozzle in sections 1 and 2.

3. By substituting the obtained values in equation (4.5), we find the corresponding values of $\lambda_{г2}$.

4. By solving equation (4.7) we find the unknown value of pressure on the edge of the liquid nozzle, after which we determine all remaining parameters of the streams on the edge of the nozzles, which are needed to find the parameters of the mixture at the ejector outlet.

Case $\sigma_{**}>\sigma>\sigma^{**}$.

In this case in the critical regime the rated flow pattern ($\lambda_{г1} = \lambda_{г.р} > 1$, $W_{п.ж1} = W_{п.ж.р} > a_{п.ж1}$) is realized in both supersonic nozzles. The flow rates of the working bodies and the parameters of the streams in the inlet section of the mixing chamber are in this case determined from calculating the flow in the nozzles (see above).

Case $\sigma^{**}>\sigma>\sigma_{min}$

1. We find the parameters of the supersonic flow of saturated vapor on the edge of the nozzle.

2. We assign a series of static-pressure values $p_{п.ж2}$ and using the saturated-vapor tables we find the corresponding parameters of the supersonic vapor flow in the cut-off section.

3. From equation (4.5) we calculate the value of the reduced velocity of the gas $\lambda_{г1}$ on the edge of the nozzle.

4. From the values cf $\lambda_{\Gamma 1}$ which we have found, using the additional conditions which determine the flow of gas in the nozzle for the supersonic range of σ variation, we find the quantities $\nu_{\Gamma 1 H}$, G_{Γ} and K.

5. Using the flow parameters which we have found, by means of equation (4.7) we find quantity $p_{п.ж2}$, which corresponds to the critical operational mode of the ejector, and then, according to parts 2, 3, and 4, we determine all the parameters of the gas flow on the edge of the nozzle.

The flow patterns shown in Fig. 9 are realized in the critical regimes of gas-liquid and liquid-gas ejectors, when both nozzles are divergent and supersonic.

In a case where the gas nozzle is divergent and the liquid nozzle convergent, at values of σ between σ_{max} and σ^*, the flow for the entire length of the liquid nozzle will be subsonic, and in sections 1 and 2 either liquid or saturated vapor can flow. When $\sigma^* \geq \sigma > \sigma_{min}$ the same flow patterns develop as shown in Fig. 9i-9o, except that the outflow velocity from the liquid nozzle, depending on the magnitude of $p_{ж0H}$, can be equal either to W_{s1} or $a_{п.ж1}$.

In an ejector with a convergent gas nozzle and a divergent liquid nozzle when $\sigma_{max} \geq \sigma > \sigma^*$ the flow patterns presented in Fig. 9a-9i develop, and the velocity at the edge of the gas nozzle is equal to the critical speed of sound ($\lambda_{\Gamma 1} = 1$); when $\sigma^* \geq \sigma > \sigma_{min}$ in the initial part of the mixing chamber the supersonic stream of saturated vapor expands and the gas stream, whose velocity for the entire length of the nozzle is subsonic, contracts.

In a case where both nozzles are convergent there occurs a sudden expansion of the sonic gas stream and a contraction of the subsonic stream flowing from the liquid nozzle in the range of

σ variation from σ_{max} to σ^* in the initial part of the mixing chamber; the additional conditions determining the critical regime will in this case be: $\lambda_{г1} = 1$; $W_{ж2} = W_{s2}$ or $W_{п.ж2} = a_{п.ж2}$.

When $\sigma^* > \sigma > \sigma_{min}$ in the initial part of the mixing chamber there occurs a sudden expansion of the sonic ($W_{п.ж1} = a_{п.ж1}$) or the supersonic ($W_{s1} > a_{п.ж1(x \to 0)}$) stream of saturated vapor and a contraction of the subsonic stream of gas, whose reduced velocity in the cut-off section becomes equal to unity ($\lambda_{г2} = 1$).

The critical regimes in all of these cases are calculated just as in the case examined above for the two supersonic nozzles.

Let us note in conclusion that a number of assumptions made in the work during derivation of the equations of critical regime for a two-phase ejector require experimental varification. In the case where between sections 1-2 a liquid flows (see Fig. 9a, b, c, d, f), this refers primarily to the proposal that the flow of liquid is in a stream (it is possible that under certain conditions the flow of liquid will break up into drops before section 2).

In the case where in the nozzle or the initial section of the mixing chamber (particularly under conditions of $\sigma < \sigma^*$) spontaneous evaporation of the liquid occurs, this is related to the assumption of thermodynamic and mechanical equilibrium of the saturated vapor which has formed; actually, due to the lag in the evaporation process, the pressure of the vapor can be lower than the saturation pressure, and thus the liquid will be overheated.

Note also that in addition to the above cases of a two-phase gas-liquid or liquid-gas ejector working in critical modes, in the liquid-gas ejector another case is possible - the case where the flow of both streams at the edge of the nozzles is subsonic. This case will be thoroughly examined in the following chapter.

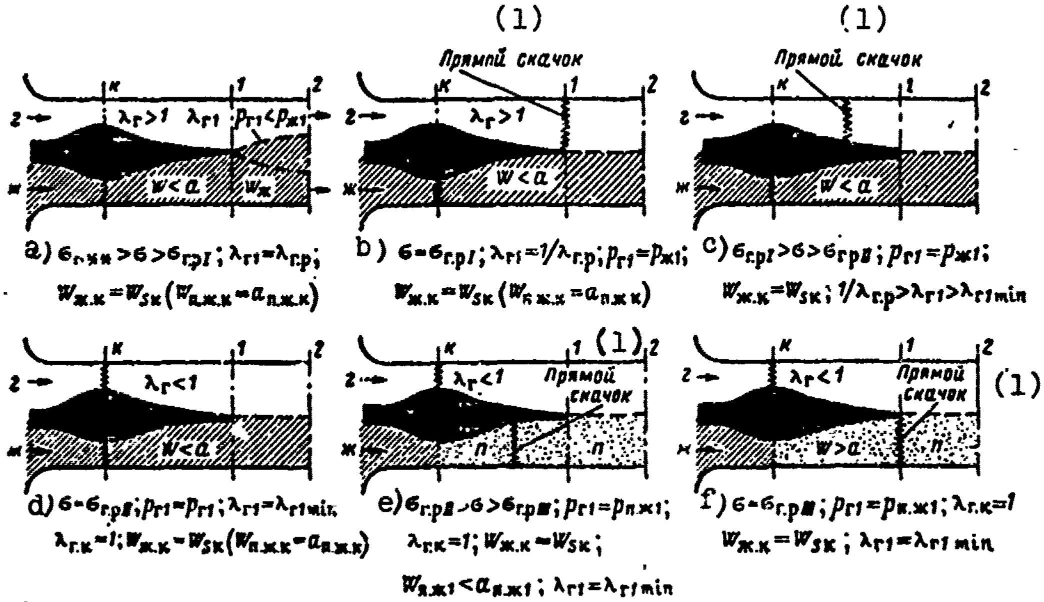
4.3. Cut-Off Regimes of Nozzles

When the ejector is operating in critical regimes, when the characteristic pressure ratio lies between $\sigma_{max}-\sigma_{***}$ (see Fig. 9b) and $\sigma^{***}-\sigma_{min}$ (see Fig. 9n), the flow of one of the working bodies for the entire length of the nozzle and the initial part of the mixing chamber will be subsonic. Thus, with an increase in counterpressure, as compared to the counterpressure value corresponding to the limiting critical regime, the ejection coefficient immediately begins to change and the ejector shifts to a subcritical working regime. The limiting critical regime in this case is simultaneously the limiting cut-off regime of the ejector (point 3 in Fig. 8a, b, d, e).

A different pattern is observed when $\sigma_{***} > \sigma > \sigma^{***}$ (see Fig. 9d-9*l*). In this range of σ variation in the critical regimes in the divergent part of both nozzles there are regions of supersonic flow, through which perturbations cannot penetrate into the ejector inlet. With an increase in counterpressure, as compared to the counterpressure value corresponding to the limiting critical regime, the flow rate of the mixed media do not vary and the ejector shifts to cut-off regime in the nozzles (section 3-4 of the vertical branches of the curves in Fig. 8c, f). In these regimes in the divergent part of at least one of the nozzles there develops a plane shock, which as counterpressure increases shifts toward the critical section. Finally, at a certain counterpressure value the limiting cut-off regime of the nozzle develops, in which the plane shock reaches the critical section of the nozzle (points 4 in Fig. 8c, f). With a further increase in counterpressure subcritical regimes develop.

Possible flow schemes in the nozzles and in the initial part of the mixing chamber for the limiting cut-off nozzle regime when $p_{ж0н} \geq p_{ж0II}$ are shown in Fig. 11.

When $\sigma_{***} > \sigma > \sigma_{г.рI}$ in the divergent part of the gas nozzle a supersonic flow ($\lambda_{г1} = \lambda_{г.р}$) is realized, and in the divergent part of the liquid nozzle, which operates in a cut-off regime ($W_{ж.к} = W_{S\ к}$, or $W_{п.ж.к} = a_{п.ж.к}$) a subsonic nozzle is realized, where, depending on the magnitude of the characteristic pressure relationship, $p_{г1} > p_{ж1}$ or $p_{г1} < p_{ж1}$ (see Fig. 11a).



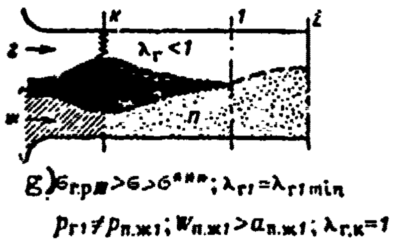


Fig. 11. Possible flow schemes in nozzles and in initial part of mixing chamber for limiting cut-off regime of nozzles in a case where $p_{ж0н} \gtrsim p_{ж0II}$.
KEY: (1) Plane shock.

With a decrease in the value of σ from σ_{***} to $\sigma_{г.рI}$ and $\sigma_{г.рII}$ the flow pattern in the liquid nozzle does not change. When $\sigma = \sigma_{г.рI}$ a plane shock develops on the edge of the gas nozzle ($\lambda_{г1} = 1/\lambda_{г.р}$, see Fig. 11b).

When $\sigma_{г.рI} > \sigma > \sigma_{г.рII}$ the plane shock is found in the divergent part of the gas nozzle (see Fig. 11c) and when $\sigma = \sigma_{г.рII}$ is in its critical section (see Fig. 11d). In the range of σ variation from $\sigma_{г.рII}$ to σ^{***} the flow in the gas nozzle remains

invariable; in the divergent part of the liquid nozzle when $\sigma_{\text{г.pII}} > \sigma > \sigma_{\text{г.pIII}}$ a plane shock develops (see Fig. 11e), which when $\sigma = \sigma_{\text{г.pIII}}$ is located in its outlet section (see Fig. 11f). When $\sigma_{\text{г.pIII}} > \sigma > \sigma^{***}$ (see Fig. 11g) a supersonic stream of saturated vapor ($W_{\text{п.ж1}} = W_{\text{п.ж.p}} > a_{\text{п.ж1}}$) flows from the liquid nozzle.

Let us find the values $\sigma_{\text{г.pI}}$, $\sigma_{\text{г.pII}}$ and $\sigma_{\text{г.pIII}}$, assuming that the physical constants of the gas and the liquid, as well as the state parameters of the liquid are assigned, and quantity $p_{\text{г0н}}$ is variable.

The streams which flow from the nozzles in regimes $\sigma = \sigma_{\text{г.pI}}$, $\sigma = \sigma_{\text{г.pII}}$ and $\sigma = \sigma_{\text{г.pIII}}$ are subsonic, and thus static pressures in them are identical.

When $\sigma = \sigma_{\text{г.pI}}$ from condition $p_{\text{г1}} = p_{\text{ж1}}$, if we use expression (3.15) and assume that $\lambda_{\text{г1}} = 1/\lambda_{\text{г.p}}$ and $\nu_{\text{г1н}} = \nu_{\text{г1н.p}}$, then we get

$$\sigma_{\text{г.pI}} = \frac{2}{\varkappa_{\text{г}} + 1} \frac{p_{\text{ж1}\max}}{p_{\text{ж0н}}} \frac{1}{\tilde{f}_{\text{г.к}p}\left(\frac{1}{\varphi_{\text{г.к.н}}}\right) f\left(\frac{1}{\lambda_{\text{г.p}}}\right)\lambda_{\text{г.p}}}, \qquad (4.21)$$

where the value of $p_{\text{ж1}\max}$ in a case where the compressibility of the liquid can be ignored is found from formula (3.47) using expression (3.45).

For $\sigma_{\text{г.pII}}$ from condition $p_{\text{г1}} = p_{\text{ж1}}$, using the relationship of (3.19), we get the following expression:

$$\sigma_{\text{г.pII}} = \frac{2}{\varkappa_{\text{г}} + 1} \frac{p_{\text{ж1}\max}}{p_{\text{ж0н}}} \frac{\lambda_{\text{г1}\min}}{\tilde{f}_{\text{г.к}p}\left(\frac{1}{\bar{\varphi}_{\text{г.к.н}}}\right) f(\lambda_{\text{г}\min})}, \qquad (4.22)$$

where $\lambda_{\text{г1}\min}$ and $p_{\text{ж1}\max}$ are found from (3.18) and (3.47).

The quantity $\sigma_{г.pIII}$ can also be found from relationship (4.22) if in place of $p_{ж1\ max}$ is the static pressure $p_{п.ж1}$ or $p_{ж1}$ behind the plane shock, located at the edge of the liquid nozzle, where the velocity of the supersonic flow of saturated vapor in front of it is equal to $W_{п.ж.p}$.

Now let us assign the calculation order of the flow parameters in the inlet section of the mixing chamber for an ejector working in a limiting cut-off nozzle regime for different assignments of σ.

If the assigned value of σ lies between σ_{***} and $\sigma_{г.pI}$, then the parameters of the liquid at the edge of the nozzle are found from formulas (3.45), (3.46), and (3.47), where quantities G_{max}, $\nu_{ж.к.н}$, and $W_{s\ к}$ are determined by calculating the flow in the convergent part of the nozzle in the cut-off regime. For a gas flow in this case we have

$$\lambda_{г1}=\lambda_{г.p} \text{ and } \nu_{г1н}=\nu_{г1н.p}.$$

In the case of $\sigma_{г.pI} > \sigma > \sigma_{г.pII}$ the flow parameters on the edge of the liquid nozzle are found just as in the preceding case.

To determine the parameters of the gas flow on the edge of the nozzle a series of values are assigned for reduced velocity $\lambda_{г1}$ ranging from $1/\lambda_{г.p}$ to $\lambda_{г1\ min}$. From formula (3.17) we find corresponding values for the coefficient of pressure recovery $\nu_{г1н}$, and then the values of static pressure $p_{г1} = p_{г0н}\nu_{г1н}p(\lambda_{г1})$. The values sought for $\lambda_{г1}$ and $\nu_{г1н}$ are found thereafter from the condition $p_{г1} = p_{ж1}$.

In a case where the defined value of σ lies between $\sigma_{г.pII}$ and $\sigma_{г.pIII}$ ($\lambda_{г1} = \lambda_{г1\ min}$), quantity $\nu_{г1н}$ and static pressure $p_{г1} = p_{г1\ max}$ in the limiting cut-off regime of the ejector are found from (3.18), (3.17), and (3.19). The flow parameters on the edge of the liquid nozzle are found from calculating the flow in the nozzle in the presence of a plane shock in its divergent

part from condition $p_{ж1} = p_{г1}$ or $p_{п.ж1} = p_{г1}$. The calculation is performed as follows:

1) the parameters of state and velocity of the flow in a critical section of the liquid nozzle in its cut-off regime are found, and then quantity $G_{ж\ max}$ is calculated;

2) assuming that a liquid flows from the nozzle ($x_1 = 0$), then from formula (3.45) we find the outflow velocity $W_{ж1}$, and then from expression (3.46), assuming that $p_{ж1} = p_{г1}$, we find the quantity $p_{ж01}$, after which from expression (3.30b) we find the temperature of the liquid $T_{ж1}$. If quantity $p_{г1}$ exceeds p_{s1} at $T_{ж1}$, then the calculation ends here;

3) if $p_г < p_{s1}$, then from the nozzle flows saturated vapor, whose state parameters and outflow velocity are found as follows. From the assigned value $p_{п.ж1} = p_{г1}$ from the saturated-vapor tables we find the quantities v_1', v_1'', i_1', i_1'', after which from expressions (3.41) and (3.44) we find the quantities $i_{п.ж1}$, $v_{п.ж1}(\gamma_{п.ж1})$ and from formula (3.40) we find the value $W_{п.ж1}$.

When $\sigma_{г.рIII} > \sigma > \sigma^{***}$ the parameters on the edge of the gas nozzle are determined just as in the preceding case. Here the liquid nozzle works in the calculated supersonic outflow regime ($W_{п.ж1} = W_{п.ж.р} > a_{п.ж1}$).

In the particular case, when the gas nozzle is divergent, and the liquid nozzle convergent, the cut-off regimes of the nozzles can develop in a range of σ variation from $\sigma_{г.рI}$ to $\sigma_{г.рII}$. Quantity $\sigma_{г.рI}$ and $\sigma_{г.рII}$ are found from condition $p_{г1} = p_{ж1}$, where in both cases the liquid nozzle works in the cut-off regime ($W_{ж1} = W_{s1}$ or $W_{п.ж1} = a_{п.ж1}$). In this case, when the gas nozzle is convergent and the liquid nozzle divergent, the cut-off regimes of the nozzles are realized at values of σ which lie in the range

of $\sigma_{г.pII} > \sigma > \sigma_{г.pIII}$; in determining value $\sigma_{г.pII}$ and $\sigma_{г.pIII}$ we assume that $p_{г1} = p_{ж1}$ and $\lambda_{г1} = 1$.

In an ejector with convergent nozzles the cut-off regimes of the nozzles can develop in a case where $\bar{\delta} > 0$. In the limiting cut-off regime of the nozzles in this case $\lambda_{г1} = 1$, $W_{ж1} = W_{s1}$ or $W_{п.ж1} = a_{п.ж1}$ (which depends on the magnitude of $p_{ж0н}$), while pressure p_1 in the stagnant regions, which develop near the dull edges of the nozzles, is equal to the lesser of pressures ($p_{г1}$ or $p_{ж1}$) on the edge of the nozzle. The cut-off regimes of the nozzles in this case are realized in a range of σ variation from σ_1, at which $p_{г1} > p_{ж1}$, to σ_2, at which $p_{г1} < p_{ж1}$. Quantities σ_1 and σ_2 are found from the equations of the critical regime (4.5), (4.7), and (2.10), respectively, under conditions

$$\lambda_{г1}=1;\ W_{п.ж2}=W_{п.ж1}=a_{п.ж1}\,(W_{ж2}=W_{ж1}=W_{s1})$$
$$\text{and } W_{п.ж1}=a_{п.ж1}\,(W_{ж1}=W_{s1});\ \lambda_{г2}=\lambda_{г1}=1.$$

4.4. Subcritical Regimes

Let us examine a two-phase ejector operating in subcritical regimes, where with a change in counterpressure there is a corresponding change in the flow rate of the mixed media and, consequently, in the ejection coefficients. In these regimes, just as the critical regime studied above and the cut-off regimes of the nozzles, depending on the magnitude of the characteristic pressure ratio σ, various flow patterns can develop in the nozzles and in the initial part of the mixing chamber (Fig. 12). Analysis of these patterns enables us to find the conditions which relate the flow parameters on the edge of the nozzles.

At values of σ which exceed the quantity $\sigma_{г.pIж0}$, a supersonic flow ($\lambda_{г1} = \lambda_{г.p}$, see Fig. 12a) is realized in the divergent part of the gas nozzle over the entire range of possible change in the ejection coefficient from $K = \infty$ ($p_{ж1} = p_{ж0н}$) to $K = K_3$, which corresponds to the cut-off regime of the ejector.

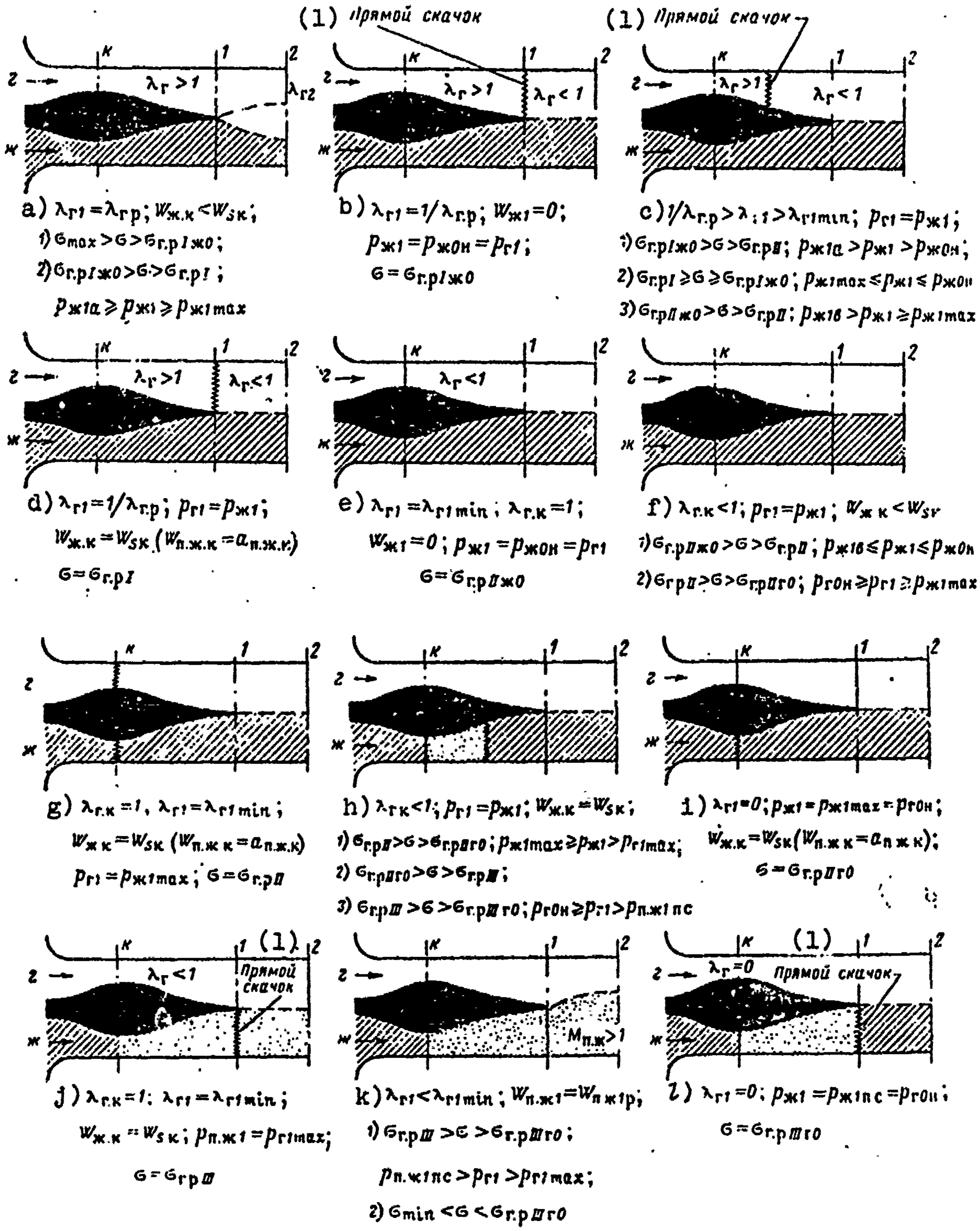


Fig. 12. Possible flow schemes in nozzles and in initial part of mixing chamber in subcritical regimes.
KEY: (1) Plane shock.

The quanity $\sigma_{г.pIж0}$, which corresponds to the regime in which the flow pattern shown in Fig. 12b is realized, can be found from formula

$$\sigma_{г.pIж0}=\frac{2}{\varkappa_г+1}\frac{1}{\tilde{f}_{г.к}p\left(\frac{1}{\varphi_{г.к.н}}\right)T\left(\frac{1}{\lambda_{г.р}}\right)\lambda_{г.р}}. \qquad (4.23)$$

At values of σ which lie between $\sigma_{г.pIж0}$ and $\sigma_{г.pI}$ (see Fig. 12d), which is determined from formula (4.21), depending on pressure on the edge of the liquid nozzle two different flow schemes can devleop. In a variation range of $p_{ж1}$ from $p_{ж1\,max}$ [relationship (3.47)] to $p_{ж1a}$, which is determined from formula

$$p_{ж1a}=\frac{\varkappa_г+1}{2}\tilde{f}_{г.к}p\left(\frac{1}{\varphi_{г.к.н}}\right)\sigma p_{ж0н}T\left(\frac{1}{\lambda_{г.р}}\right)\lambda_{г.р}, \qquad (4.24)$$

a supersonic stream ($\lambda_{г1} = \lambda_{г.р}$, see Fig. 12a) flows from the gas nozzle.

When $p_{ж1a} > p_{ж1} > p_{ж0н}$ in the divergent part of the gas nozzle a plane shock develops (see Fig. 12c); subcritical regimes are in this case calculated by means of (3.17).

When $\sigma_{г.pI} > \sigma \geq \sigma_{г.pIIж0}$ the flow scheme shown in Fig. 12c is realized over the entire range of possible change in the ejection coefficient ($\infty > K > K_3$). In this case the subcritical regimes are also calculated by means of (3.17). Quantity $\sigma_{г.pIIж0}$ (see Fig. 12e) can be found from formula

$$\sigma_{г.pIIж0}=\frac{2}{\varkappa_г+1}\frac{\lambda_{г1\,min}}{\tilde{f}_{г.к}p\left(\frac{1}{\varphi_{г.к.н}}\right)T(\lambda_{г1\,min})}. \qquad (4.25)$$

Depending on quantity $p_{ж1}$, two flow schemes correspond to the values of σ which lie between $\sigma_{г.pIIж0}$ and $\sigma_{г.pII}$ (see Fig. 12g), which is determined by (4.22). With a change in pressure on the edge of the liquid nozzle from $p_{ж1\,max}$ [relationship (3.47)] to $p_{ж1в}$, determined from formula

$$p_{ж1в}=\frac{\varkappa_г+1}{2}\tilde{f}_{г.к}p\left(\frac{1}{\varphi_{г.к.н}}\right)\sigma p_{ж0н}\frac{T(\lambda_{г1min})}{\lambda_{г1min}}, \qquad (4.26)$$

the flow scheme shown in Fig. 12c develops. With a change in pressure on the edge of the liquid nozzle from $p_{ж1в}$ to $p_{ж0н}$ over the entire length of both nozzles the flow is subsonic (see Fig. 12f). An additional condition enabling us to calculate these regimes will be $p_{г1} = p_{ж1}$.

When $\sigma_{г.pII} > \sigma > \sigma_{г.pIIг0}$ with a decrease in the static pressure of the gas on the edge of the nozzle from $p_{г0н}$ to $p_{г1} = p_{ж1\,max}$ [relationship (3.47)] the flow in both nozzles is subsonic (see Fig. 12f). With a further decrease in quantity $p_{г1}$ from $p_{г1} = p_{ж1\,max}$ to $p_{г1\,max}$ [relationship (3.19)] in the divergent part of the liquid nozzle a supersonic flow of saturated gas develops which is shut off by the plane shock (see Fig. 12h).

The flow pattern for the regime $\sigma = \sigma_{г.pIIг0}$ is shown in Fig. 12i in this regime $p_{г1} = p_{г0н}$ ($\lambda_{г1} = 0$) and $p_{ж1} = p_{ж1\,max}$. Quantity $p_{г.pIIг0}$ is found from the obvious relationship:

$$\sigma_{г.pIIг0}=p_{ж1max}/p_{ж0н}. \qquad (4.27)$$

Quantity $p_{ж1\,max}$, contained in this relationship, is found from (3.47).

The flow scheme shown in Fig. 12h is also realized when $\sigma_{г.pIIг0} > \sigma > \sigma_{г.pIII}$ over the entire range of change in the ejection coefficient (from $K = \infty$, $p_{г1} = p_{г0н} = p_{ж1}$ to K_3, $p_{г1} = p_{г1\,max}$). The flow pattern for regime $\sigma = \sigma_{г.pIII}$ is shown in Fig. 12j. Quantity $\sigma_{г.pIII}$ can be found from formula (4.22), where in place of $p_{ж1\,max}$ we should substitute static pressure $p_{п.ж1п.с}$ behind the plane shock, which develops in the outlet section of the liquid nozzle (see Fig. 12l).

In the range of σ variation from $\sigma_{\text{г.pIII}}$ to $\sigma_{\text{г.pIIIг0}}$, determined from formula

$$\sigma_{\text{г.pIIIг0}}=\frac{p_{\text{п.ж1п.с}}}{p_{\text{ж0н}}}, \qquad (4.28)$$

three flow schemes are possible. When $p_{\text{г0н}} \gtrsim p_{\text{г1}} > p_{\text{п.ж1п.с}}$ a flow with a plane shock develops in the liquid nozzle (see Fig. 12h) and when $p_{\text{п.ж1п.с}} > p_{\text{г1}} > p_{\text{г1 max}}$ a supersonic stream of saturated vapor flows from the liquid nozzle (see Fig. 12k). The flow pattern shown in Fig. 12k when $\sigma < \sigma_{\text{г.pIIIг0}}$ is realized for the entire possible range of change in the ejection coefficient (from $K = 0$ to $K = K_3$).

Having determined which of the possible ranges of change in characteristic pressure ratio contains the assigned value of σ, then, if we use the conditions described above and the relationships which determine the flows in the nozzles (see Chapter 3), then we can find the flow parameters on the edges of the nozzles for the entire possible range of change in the ejection coefficient and, finally, by using the ejection equation we can find dependences $p_{\text{c04}}(K)$ and $\eta(K)$.

4.5. Cut-Off Regimes of Mixing Chamber

Analysis of the ejection equation system shows that in a two-phase ejector with fixed geometry the critical regimes and the cut-off regimes of the nozzles can only be realized in a certain range of change in the parameters of state of the gas and the liquid at the nozzle inlet. Within this range the ejection equations for conditions at the inlet to the mixing chamber, which correspond to critical regimes and cut-off regimes of the nozzles, do not have solutions: the subradical expression in (2.18), which determines the velocity of the mixture in the outlet section of the mixing chamber, becomes negative.

The cut-off regime of the ejector is in this case the cut-off regime of the mixing chamber, in which the subradical expression in relationship (2.18) reverts to zero, which corresponds to fulfillment of condition $W_{c3} = W''_{c3} = a_{c3}$ ($M_{c3} = 1$). With a decrease in counterpressure from the value corresponding to regime K = 0 or K = ∞ (point 6 in Fig. 8), in this case even before the advent of the critical regime (point 3 in Fig. 8) or the cut-off regime of the nozzles (point 4 in the same figure) the mixing chamber (point 5') can be cut off, and thus with a further decrease in counterpressure quantity K becomes invariable, while the total pressure of the mixture decreases accordingly (sections 5'-7 of the curves in Fig. 8).

The cut-off regime of the mixing chamber can also develop in a range of counterpressure variation corresponding to the cut-off regimes of the nozzles (point 5' in Fig. 8f). The boundary of the region of cut-off regimes of the mixing chamber corresponds to the values of the state parameters of the gas and the liquid at the ejector inlet at which the conditions of the critical regime or the cut-off regime of the nozzles and the cut-off regime of the mixing chamber are fulfilled simultaneously.

CHAPTER 5

SOME OPERATIONAL PECULIARITIES OF A LIQUID-GAS EJECTOR WITH CONVERGENT NOZZLES

Let us study more thoroughly the work of a liquid-gas ejector with convergent nozzles whose edges are of zero thickness.

5.1. Subcritical Regimes

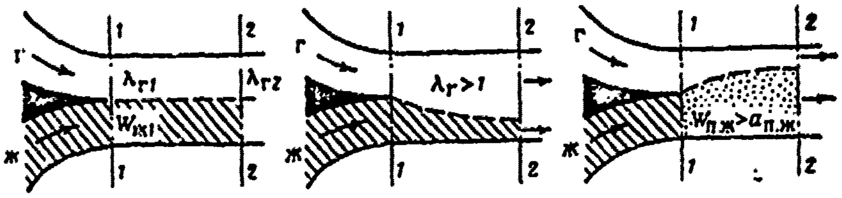
When a liquid-gas ejector with convergent nozzles operates in subcritical regimes three flow schemes are possible in the nozzles and in the initial part of the mixing chamber:

1) when both flows are subsonic;

2) when the gas nozzle is choked;

3) when the liquid nozzle is choked.

In the first case an additional condition to the system of ejection equations is the condition of equality of static pressures on the edges of the nozzles ($p_{\Gamma 1} = p_{\text{ж}1}$ or $p_{\Gamma 1} = p_{\text{п.ж}1}$), in the second case - condition $\lambda_{\Gamma 1} = 1$, and in the third - the condition $W_{\text{ж}1} = W_{s\ 1}$ or $W_{\text{п.ж}1} = a_{\text{п.ж}1}$. Possible flow schemes in the initial part of the mixing chamber for these cases are shown in Fig. 13.



a) $W_{ж1}<W_{s1}$ ($W_{п.ж1}<a_{п.ж1}$); $\lambda_{г1}<1$; $p_{г1}=p_{ж1}$
b) $W_{ж1}<W_{s1}$ ($W_{п.ж1}<a_{п.ж1}$); $\lambda_{г1}=1$; $\lambda_{г2}>1$; $p_{г1}>p_{ж1}$
c) $W_{ж1}=W_{s1}$ ($W_{п.ж1}=a_{п.ж1}$); $W_{п.ж2}>a_{п.ж2}$; $\lambda_{г1}<1$; $p_{г1}<p_{ж1}$

Fig. 13. Possible flow schemes in initial part of mixing chamber of liquid-gas ejector with convergent nozzles in subcritical regimes.

In the case where $p_{г01}p_г(1) > p_{s1}$ and $p_{г0н} < p_{ж0н}$, condition $p_{г1} = p_{ж1}$ is fulfilled in a range of variation in the outflow velocity of the liquid from the minimal value of $W_{ж1\ min}$, which corresponds to regime $\lambda_{г1} = 0$, to the maximal value of $W_{ж1\ max}$, in which $\lambda_{г1} = 1$ (see Fig. 13a); when $p_{г0н} > p_{ж0н}$ this condition is fulfilled in a range of variation in $W_{ж1}$ from zero ($\lambda_{г1\ min}$) to $W_{ж1\ max}$ ($\lambda_{г1} = 1$). At values $W_{ж1}$ which exceed $W_{ж1\ max}$ the condition $\lambda_{г1} = 1$ is fulfilled; here the sonic gas flow suddenly expands in the initial section of the mixing chamber and constricts the stream of liquid (see Fig. 13b).

In a case where $p_{г01}p_г(1) < p_{s1}$, condition $p_{г1} = p_{ж1}$ is fulfilled at values of $\lambda_{г1}$ which lie between zero (when $p_{г0н} < p_{ж0н}$) or quantity $\lambda_{г1\ min}$ (when $p_{г0н} > p_{ж0н}$), determined from condition $p_{г1} = p_{ж0н}$, and the quantity $\lambda_{г1\ max}$, which is determined when $p_{ж0н} > p_{ж0II}$ from relationship $p(\lambda_{г1\ max}) = p_{s1}/p_{г01}$. When $p_{ж0II} > p_{ж0н} > p_{s1}$, this quantity is determined from formula $p(\lambda_{г1\ max}) = p_{п.ж1\ min}/p_{г01}$, where $p_{п.ж1\ min}$ is found from condition $W_{п.ж1} = a_{п.ж1}$.

When $\lambda_{г1} > \lambda_{г1\ max}$ condition $W_{ж1} = W_{s1}$, or $W_{п.ж1} = a_{п.ж1}$ is fulfilled; in the initial part of the mixing chamber there is a sudden expansion of the supersonic stream of saturated vapor and a contraction of the subsonic stream of gas (see Fig. 13c).

5.2. Critical Regimes and Cut-Off Regimes of Mixing Chamber

Theoretical analysis of flow conditions in the mixing chamber of the ejector indicate that a supersonic flow of the mixture of gases is possible only when at least one of the flows mixed in the outlet section of the nozzles or in the initial part of the mixing chamber is supersonic. In a case where both flows at the inlet to the mixing chamber of the ejector are subsonic, a supersonic flow of the mixture is impossible, since this would contradict the second law of thermodynamics (the entropy of the mixture becomes less than the total entropies of the mixed gases). This is explained by the fact that the critical velocity of the mixture always lies between the critical velocities of the high-pressure and the low-pressure gases, and thus the transition to the supersonic region of the flow at subsonic velocities of the mixed streams is only possible as a result of the development of an expansion shock.

As indicated above, in gas-liquid and liquid-gas ejectors with divergent nozzles the supersonic flow of the mixture which corresponds to an ejector working in critical regimes, can also develop in a case where one or both of the streams in the initial part of the mixing chamber is supersonic (see Fig. 9). In the particular case of an ejector with convergent nozzles the flow schemes shown in Fig. 14 can correspond to the critical regimes, depending on the parameters of state of the gas and the liquid.

When $p_{\Gamma 01}p_{\Gamma}(1) > p_{s1}$ (see Fig. 14a) in critical regimes the velocity of the gas on the edge of the nozzle is equal to the local speed of sound ($\lambda_{\Gamma 1} = 1$), where $p_{\Gamma 1} > p_{ж1}$. In the initial part of the mixing chamber there occurs a sudden expansion of the gas stream, whose velocity in section 2 becomes supersonic ($\lambda_{\Gamma 2} > 1$). The stream of liquid between sections 1-2 is compressed and its velocity rises to the maximal possible value ($W_{ж2} = W_{s2}$ or $W_{п.ж2} = a_{п.ж2}$). When $p_{\Gamma 01}p_{\Gamma}(1) < p_{s1}$ (see Fig. 14b) in the

critical regime the liquid nozzle is cut off ($W_{ж1} = W_{s1}$, or $W_{п.ж1} = a_{п.ж1}$), and $p_{ж1} > p_{г1}$. Between sections 1-2 of the mixing chamber a supersonic stream of saturated vapor develops ($W_{п.ж2} > a_{п.ж2}$), which, as it expands, compresses the gas stream, and thus the velocity of the latter in the cut-off section becomes equal to the speed of sound.

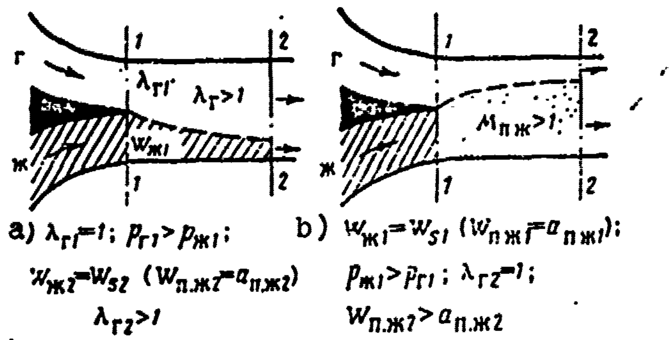


Fig. 14. Possible flow schemes in critical regimes for liquid-gas ejector with convergent nozzles.

An interesting feature of the liquid-gas ejector, which distinguishes it from the gas and gas-liquid ejectors, is the fact that under certain conditions critical regimes can develop in it at subsonic velocities of the gas and liquid flows. This is explained by the fact that the speed of sound in the two-phase gas-liquid mixture can be considerably lower than the speed of sound in its components. For example, when $t_c = 15°C$; $p_c = 0.1$ kgf/cm^2 and $K = 10^{-5}-10^{-2}$ the speed of sound in an equilibirum water-air mixture varies in a range from 5 to 30 m/s, while under the same conditions the speed of sound in water is equal to 1400 m/s, and 340 m/s in air (see Fig. 1).

The conversion of subsonic gas and liquid flows into the supersonic flow of the two-phase mixture in the mixing chamber of the ejector can therefore occur as a result of the drastic decrease in the speed of sound in the process of the formation of the

mixture, rather than from the increase in the velocities of the gas and liquid particles.

To illustrate this Figs. 15, 16, and 17 show three possible flow schemes in the mixing chamber for a liquid-gas ejector, which correspond to the ejector operating in regimes in which the velocities of the gas and liquid streams are subsonic and differ insignificantly from one another ($W_{\Gamma 1} \approx W_{ж1}$). In this case the mixing of the gas and liquid streams occurs at almost invariable values of static pressure p_c ($p_c \approx p_{\Gamma 1} = p_{ж1}$) and velocity W_c ($W_c \approx W_{ж1} \approx W_{\Gamma 1}$) along and across the mixing chamber. The speed of sound a_c and number M_c of the two-phase gas-liquid flow (hatched regions in Figs. 15-17) in this case change only in connection with the change in the relative concentration of gas in the mixture, described by the quantity $K = G_\Gamma/G_ж$.

Between sections I-III of the mixing chamber the stream of liquid is broken up into drops and a steadily expanding two-phase flow region is formed, which borders on the flow regions of the liquid and the gas (dashed lines). Between section III, in which there are no longer individual streams of gas and liquid, and the outlet section of the mixing chamber there occurs a total mixing of the gas and liquid particles and the formation of a homogeneous two-phase gas-liquid flow.

In arbitrary section II in the region between points 1 and 2 flows a supersonic gas stream ($M_\Gamma \approx M_{\Gamma 1} < 1$), while between points 5 and 6 - a subsonic liquid stream ($M_ж \approx M_{ж1} << 1$). Between points 2 and 5 flows a two-phase gas-liquid flow, and the magnitude of ratio $G_\Gamma/G_ж$ varies from ∞ on the boundary with the gas stream (point 2) to zero on the boundary with the liquid stream (point 5). Since static pressure over the section of the mixing chamber does not change, then the speed of sound in the mixture a_c with the transition from point 2 to point 5 first decreases from a value of $a_c = a_\Gamma$ to a certain minimal value of $a_{c\ min}$, and then rises to a value of $a_c = a_ж$. Number M_c first rises from $M_c = M_\Gamma$ to a certain maximal value of $M_{c\ max}$ and then decreases to $M_c = M_ж$.

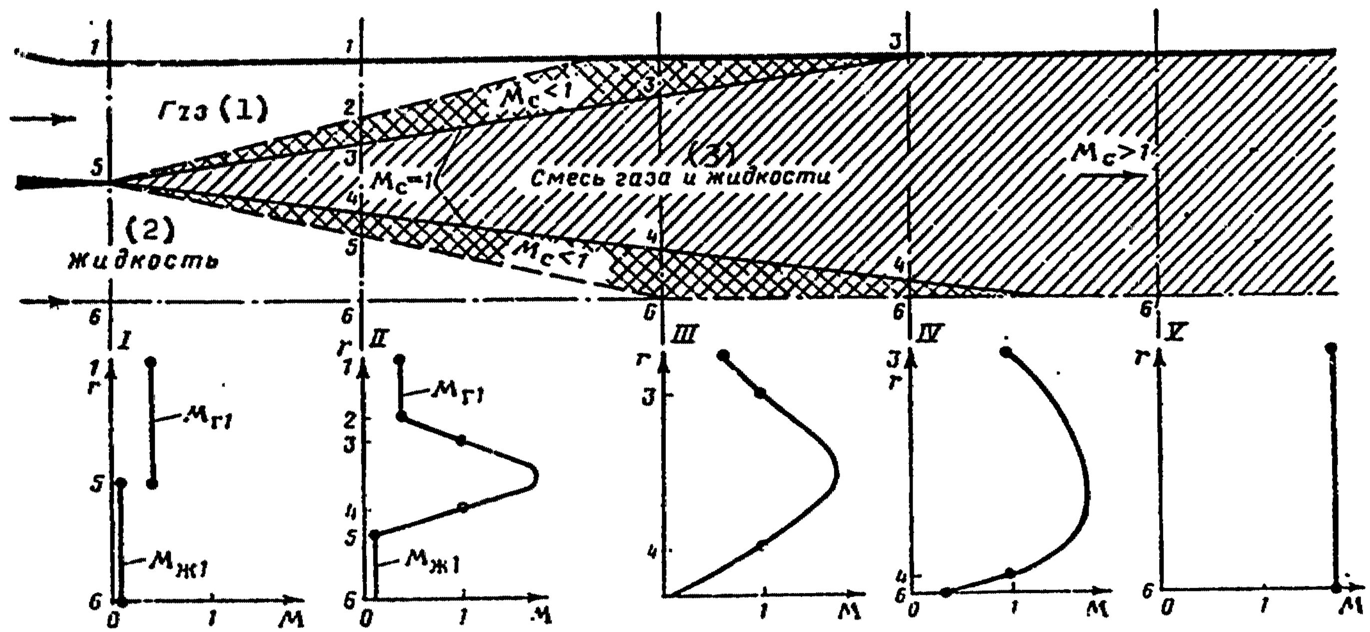


Fig. 15. One of possible flow schemes in mixing chamber of liquid-gas ejector.
KEY: (1) Gas; (2) Liquid; (3) Mixture of gas and liquid.

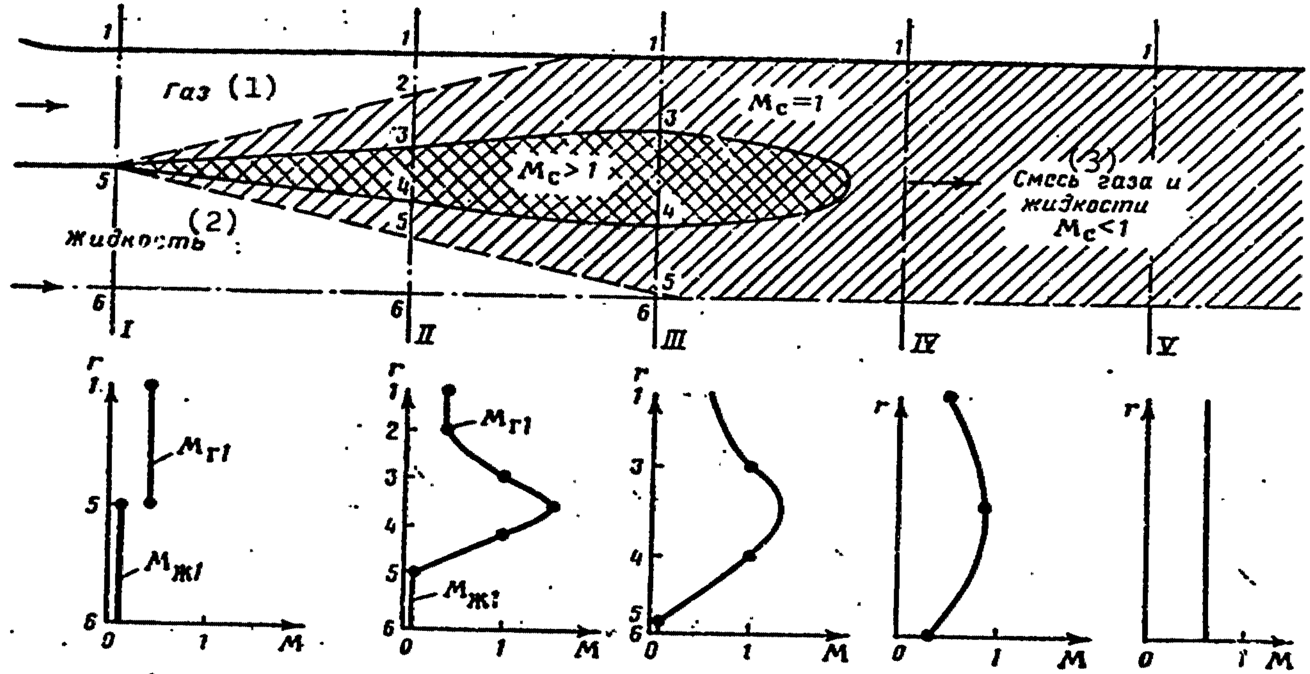


Fig. 16. One of possible flow schemes in mixing chamber of liquid-gas ejector.
KEY: (1) Gas; (2) Liquid; (3) Mixture of gas and liquid.

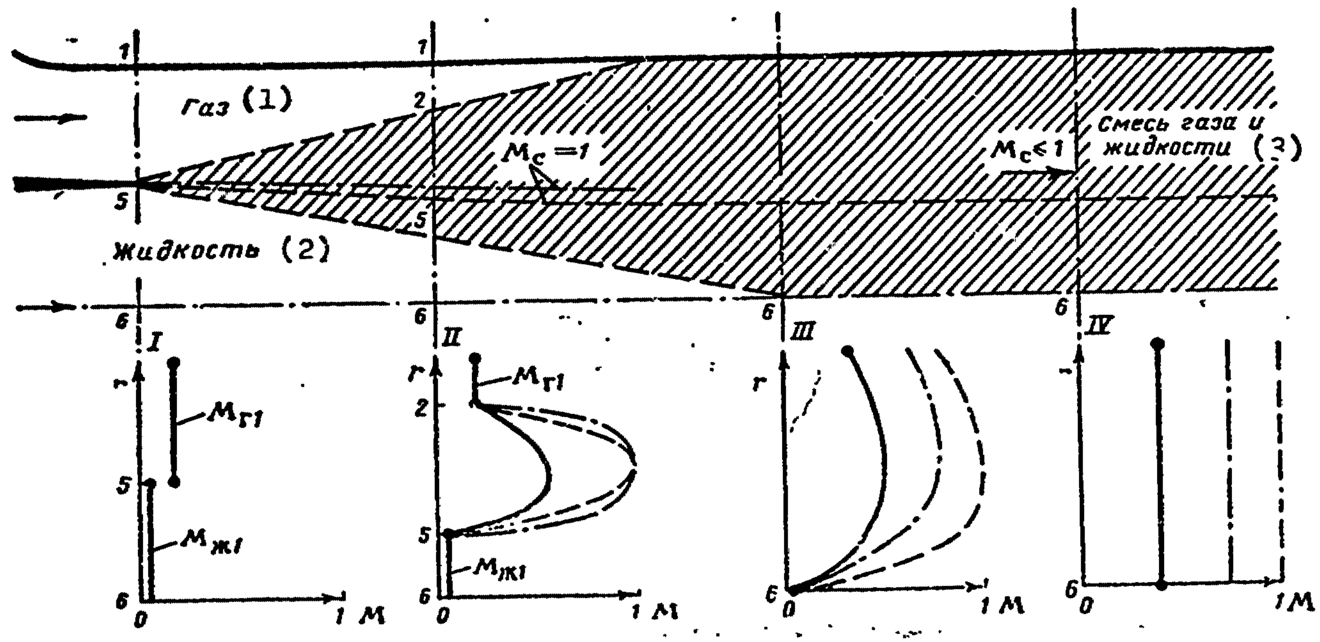


Fig. 17. One of possible flow schemes in mixing chamber of liquid-gas ejector.
KEY: (1) Gas; (2) Liquid; (3) Mixture of gas and liquid.

If the velocity of the mixture flow W_c (dashed line in Fig. 1) is less than $a_{c\ min}$ for a given value of p_c (see, for example curve $a_c(K)$ when $p_c = 10$ kgf/cm^2 in Fig. 1), then the velocity of the two-phase flow of the mixture will be less than the speed of sound over the entire range of change in ratio $G_г/G_ж$ from ∞ (points 2; 1 in Fig. 1) to zero (points 6; 5 in Fig. 1). Since quantities p_c and W_c along the mixing chamber remain virtually unchanged, then in the studied case the flow of mixture will be subsonic for the entire length (solid lines in curves of Fig. 17).

As the pressure of the mixture decreases at W_c = const this flow pattern is preserved up to the moment when quantity $a_{c\ min}$ becomes equal to W_c [curve $a_c(K)$ in Fig. 1 touches line W_c = const at point $K_{a_{c\ min}}$]. If in this case the assigned value of the ejection coefficient $K = G_{г.н}/G_{ж.н}$ does not equal $K_{a_{c\ min}}$, then, as follows from Fig. 1, the flow in the outlet part of the mixing chamber (sections III and IV in Fig. 17) will be subsonic. Here the velocity of the mixture reaches the speed of sound only in the initial part of the mixing chamber along a certain line (the dash-dot curves in Fig. 17).

In a case where $G_{г.н}/G_{ж.п} = K_{a_{c\ min}}$, the velocity of the flow in the outlet section of the mixing chamber is equal to the speed of sound, and for the entire length there exists a line (or a narrow region) on which $W_c = a_c$ (dotted curve in Fig. 17).

At even lower values of the mixture curve $a_c(G_г/G_ж)$ intersects line W_c = const at two points [curve $a_c(G_г/G_ж)$ at $p_c = 1$ kgf/cm^2 in Fig. 1]. Here, in the range of variation of ratio $G_г/G_ж$ from the value corresponding to point 3 to the value corresponding to point 4 (see Fig. 1 and also Figs. 15 and 16) the flow of the mixture is supersonic. If the assigned value of the ejection coefficient lies between quantities $(G_г/G_ж)_3$ and $(G_г/G_ж)_4$ (point C' in Fig. 1), then the range of supersonic flow of the mixture extends the entire length of the mixing chamber, so that $M_{c3} > 1$

(see Fig. 15). If, however, $K < (G_г/G_ж)_4$ or $K > (G_г/G_ж)_3$ (points C" and C"' in Fig. 1), then the range of supersonic flow exists only in the initial part of the mixing chamber, while the flow at the outlet from the mixing chamber will be subsonic (see Fig. 16). It is interesting to note that the transition from the supersonic region of the flow to the subsonic in the last case is accomplished without shocks and only as the result of a change in the distribution of the relative content of gas in the mixture.

For the purpose of showing the operational peculiarities of the liquid-gas ejector with convergent nozzles in critical regime, the results of calculating the parameters of the mixture for a water-air ejector at $\alpha = 3.3$; $\bar{\delta} = 0$; $p_{ж0н} = 5$ kgf/cm^2; $T_{ж.н} = T_{г0II} = 288°K$ and several total-pressure values of the ejected gas are presented below. For the sake of simplicity in the calculation it was assumed that in the initial part of the mixing chamber vapor from the liquid is absent, the liquid contains no dissolved gases, and the flow in the nozzles occurs without total-pressure losses. The results of calculating the parameters of the mixture at $p_{г0н} = 3$; 1.0 and 0.1 kgf/cm^2 are shown in Table 1 and in Figs. 18, 19, and 20.

If we examine Figs. 18-20 we see that at the assigned parameters of state of the gas and the liquid at the nozzle inlet and fixed geometrical parameter of the ejector the formal solution of the ejection equations provides for each value of the ejection coefficient two velocity values for the mixture (W'_{c3} and W''_{c3}) and two values for the speed of sound in the mixture (a'_{c3} and a''_{c3}), where $M'_{c3} = (W'_{c3}/a'_{c3}) > 1$, and $M''_{c3} = (W''_{c3}/a''_{c3}) < 1$ (see Table 1). With an increase in the ejection coefficient the difference between the velocities of the mixture W'_{c3}, W''_{c3} and the speeds of sound a'_{c3} and a''_{c3} decreases and, finally, at a certain value of the ejection coefficient it becomes equal to zero ($W'_{c3} = W''_{c3} = a'_{c3} = a''_{c3}$; $M'_{c3} = M''_{c3} = 1$, points a in Figs. 18-20). At values of the ejection coefficient which exceed the quantity corresponding to point a, the ejection equations do not have a solution.

Table 1. Results of calculating flow in the outlet section of a mixing chamber of a water-air ejector at α = 3.3; $p_{ж0н}$ = 5 kgf/cm^2; $T_{ж.н} = T_{г0н}$ = 288°K.

	$\lambda_{г1}$	$W_{г1}$	$W_{ж1}$	$K \cdot 10^3$	W'_{c3}	a'_{c3}	M'_{c3}	p'_{c3}, $\frac{кГ}{м^2}$	p'_{c03}, $\frac{кГ}{м^2}$	$\eta'_{нз}$	W''_{c3}	a''_{c3}	M''_{c3}	p''_{c3}, $\frac{кГ}{м^2}$	p''_{c03}, $\frac{кГ}{м^2}$	$\eta''_{нз}$	p''_{c03}/p'_{c03}
a) $p_{г01}$ = 3 кГ/см²	0,01	3,11	19,81	1,841	81,57	13,06	6,24	927	253634	—0,162	6,58	41,08	0,16	36206	37763	0,292	0,149
	0,03	9,32	19,82	5,520	76,97	22,64	3,40	2956	130844	—0,845	10,88	36,96	0,29	34177	36791	0,719	0,281
	0,05	15,53	19,83	9,188	71,74	29,31	2,45	5311	70273	—3,246	15,83	38,73	0,41	31834	35756	0,955	0,509
	0,06	19,88	19,84	11,745	67,48	33,24	2,03	7255	50422	—121,434	19,89	40,36	0,49	29907	34946	1,000	0,693
	0,07	21,74	19,85	12,836	65,43	34,82	1,88	8202	44949	8,650	21,86	40,06	0,53	28968	34559	0,991	0,769
	0,09	27,95	19,88	16,459	56,61	39,85	1,42	12325	34696	1,296	30,40	43,18	0,70	24882	33130	0,816	0,955
	0,10	31,53	19,90	18,528	43,43	43,43	1,00	18617	32022	0,566	43,43	43,43	1,00	18617	32022	0,566	1,000
b) $p_{г0н}$ = 1 кГ/см²	0,01	3,11	28,00	0,434	42,05	7,08	5,94	669	72799	—0,318	7,52	44,64	0,17	23599	26101	0,147	0,358
	0,03	9,32	28,00	1,302	39,80	12,39	3,21	2145	48811	14,636	9,74	31,28	0,31	22125	25488	0,419	0,522
	0,05	15,53	28,00	2,169	37,16	16,22	2.29	3885	34485	1,458	12,36	28,30	0,44	20388	24691	0,653	0,716
	0,07	21,74	28,01	3,033	33,82	19,59	1,73	6099	26745	1,081	15,68	27,06	0,58	18183	23820	0,848	0,891
	0,09	28,02	28,02	3,901	28,00	23,37	1,20	9972	22838	1,000	21,48	25,74	0,83	14322	22729	0,990	0,995
	0,09	28,95	28,02	4,030	24,74	24,74	1,00	12148	22535	1,005	24,74	24,74	1,00	12148	22535	1,005	1,000
c) $p_{г0н}$ = 0,1 кГ/см²	0,01	3,11	31,00	0,039	32,24	2,32	13,89	95	50976	—1,331	7,34	101,91	0,07	18395	21099	0,035	0,414
	0,03	9,32	31,01	0,118	31,97	4,02	7,95	288	47344	1,438	7,60	60,40	0,13	18202	21005	0,104	0,444
	0,07	21,74	31,01	0,274	31,43	6,17	5,09	687	41567	1,020	8,15	41,45	0,20	17804	20823	0,240	0,501
	0,10	31,06	31,01	0,390	31,00	7,40	4,19	997	38023	1,000	8,57	35,88	0,24	17495	20687	0,340	0,544
	0,30	93,18	31,02	1,132	27,85	13,05	2,13	3338	24038	1,169	11,75	25,15	0,47	15189	19734	0,941	0,821
	0,50	155,3	31,05	1,760	23,52	17,40	1,35	6574	19047	1,371	16,14	21,82	0,74	12017	18783	1,352	0,986
	0,36	175,49	31,06	1,929	19,84	19,84	1,00	9308	18365	1,496	19,84	19,84	1,00	9308	18365	1,495	1,000

Designation: кГ/см^2 = kgf/cm^2.

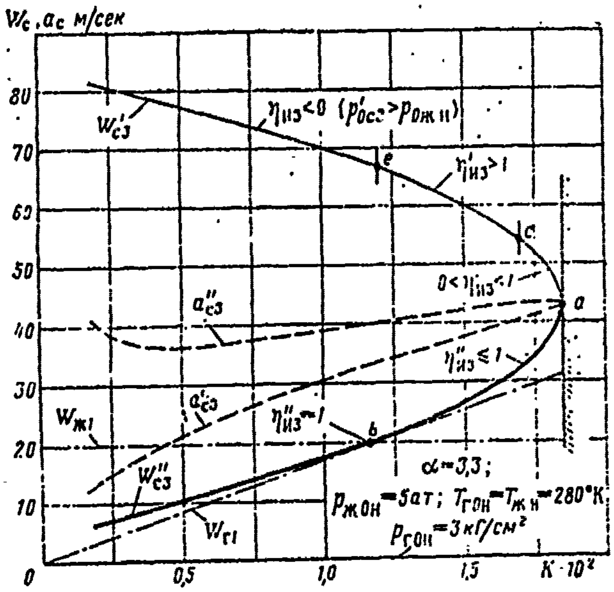


Fig. 18. Results of calculating parameters of mixture at outlet from mixing chamber of water-air ejector at $\alpha = 3.3$; $\bar{\delta} = 0$; $p_{ж0н} = 5$ kgf/cm^2; $T_{ж.н} = T_{г0н} = 280°K$ and $p_{г0н} = 3$ kgf/cm^2.

Designations: м/сек = m/s; кГ/см^2 = kgf/cm^2; ат = at.

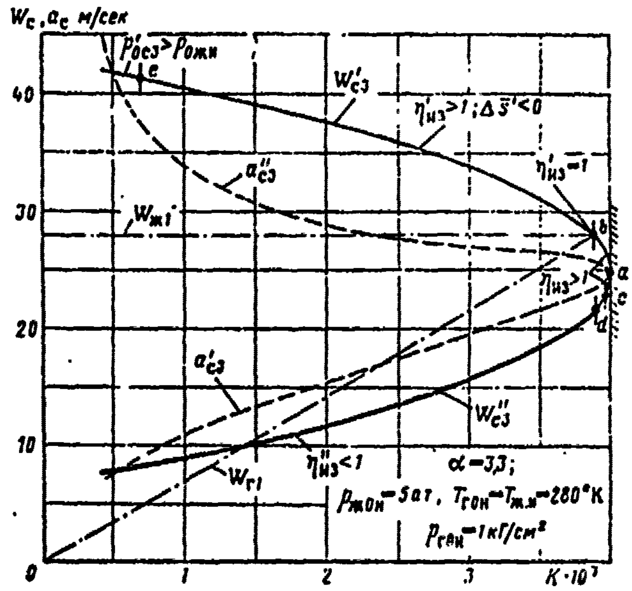


Fig. 19. Results of calculating parameters of mixture at outlet from mixing chamber of water-air ejector at $\alpha = 3.3$; $\bar{\delta} = 0$; $p_{ж0н} = 5$ kgf/cm^2; $T_{ж.н} = T_{г0н} = 280°K$ and $p_{г0н} = 1.0$ kgf/cm^2.

Designations; м/сек = m/s; кГ/см^2 = kgf/cm^2; ат = at.

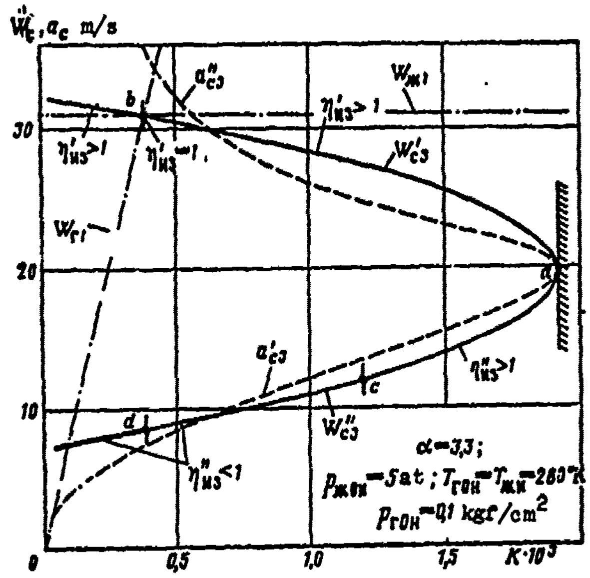


Fig. 20. Results of calculating parameters of mixture at outlet from mixing chamber of water-air ejector at $\alpha = 3.3$; $\bar{\delta} = 0$; $p_{жOH} = 5$ kgf/cm^2; $T_{ж,H} = T_{ГOH} = 280°K$ and $p_{ГOH} = 0.1$ kgf/cm^2.

As the total pressure of the ejected gas decreases from 3 to 0.1 kgf/cm^2 the maximal possible value of the ejection coefficient, which corresponds to the cut-off regime of the mixing chamber, decreases steadily - from 0.0185 to 0.0019, while the velocity of the mixture, which is equal to the critical speed of sound, decreases from 43.43 to 19.84 m/s. From Figs. 18-20 it follows that when $p_{ГOH}$ = const the velocity of the liquid at the edge of the nozzle is virtually independent of the ejection coefficient, while the velocity of the gas with an increase in the ejection coefficient from zero to K_3 rises according to a nearly linear law from zero to $W_{Г1\ max}$, which exceeds the velocity of the liquid.

At a certain value of K, which depends on the total pressure of the gas $p_{ГOH}$, the velocities of the gas and the liquid at the edge of the nozzle becomes the same and equal to the velocity of the mixture in the outlet section of the mixing chamber (point b

in Figs. 18-20). When $p_{\Gamma 0H} > 1$ kgf/cm^2 point b lies on curve $W''_{c3}(K)$, which corresponds to the subsonic flow of the mixture ($W_{\Gamma 1} = W_{ж1} = W''_{c3}$), and when $p_{\Gamma 0H} \leq 1$ kgf/cm^2, it lies on curve $W'_{c3}(K)$, which corresponds to the supersonic flow of the mixture ($W_{\Gamma 1} = W_{ж1} = W'_{c3}$). As total pressure of the ejected gas decreases from 3 to 0.1 kgf/cm^2 the speed of sound a_{c3} in regime $W_{\Gamma 1} = W_{ж1} = W_{c3}$ decreases, and the velocity of the mixture W_{c3} in the outlet section of the mixing chamber increases. Thus, the number M_{c3} increases steadily (from $M_{c3} = M''_{c3} = 0.49$ at $p_{\Gamma 0H} = 3$ kgf/cm^2 to $M_{c3} = 1$ at a value of $p_{\Gamma 0H}$ which somewhat exceeds 1 kgf/cm^2, and to $M_{c3} = 4.19$ at $p_{\Gamma 0H} = 0.1$ kgf/cm^2) (see Table 1).

In order to determine the physically possible operational regimes of the ejector Figs. 21 and 22 show dependences $\eta''_{из}(K)$ and $\eta'_{из}(K)$ for a number of values of $p_{\Gamma 0H}$ and regions where $\eta_{из} > 0$, obtained from the same calculation. At values of $p_{\Gamma 0H}$ which are equal to 3 and 4 kgf/cm^2, as follows from Fig. 21 and Table 1, a subsonic flow of the mixture at the outlet from the mixing chamber is physically possible for the entire range of variation in the ejection coefficient from zero to K_3, since at no place does quantity $\eta''_{из}$ exceed unity.

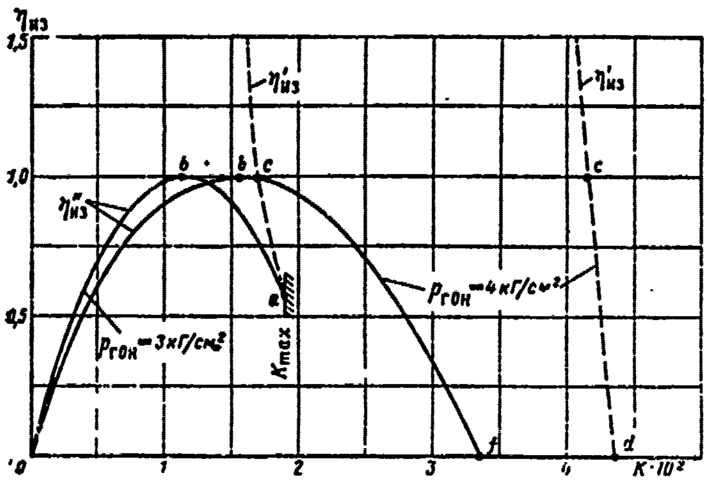


Fig. 21. Dependences of $\eta''_{из}(K)$ and $\eta'_{из}(K)$ for water-air ejector at $p_{\Gamma 0H} = 3$ kgf/cm^2 and $p_{\Gamma 0H} = 4$ kgf/cm^2. Designation: кГ/см^2 = kgf/cm^2.

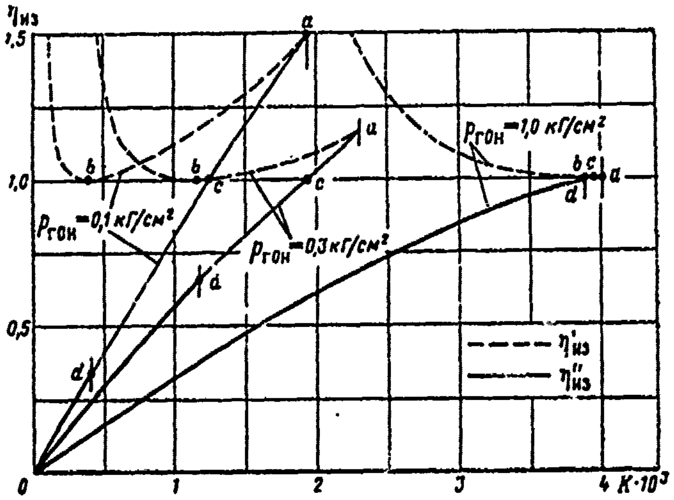


Fig. 22. Dependences $\eta''_{из}(K)$ and $\eta'_{из}(K)$ for water-air ejector at values of $p_{г0н}$ equal to 0.1, 0.3, and 1.0 kgf/cm^2. Designation: кГ/см^2 = kgf/cm^2.

With an increase in the ejection coefficient from zero to the value corresponding to regime $W_{г1} = W_{ж1} = W''_{c3}$ (point b), the efficiency of the ejector steadily increases from zero to unity, but with a further increase in the ejection coefficient quantity $\eta''_{из}$ decreases to a certain minimal value corresponding to the cut-off regime of the mixing chamber (point a). In a case where $p_{г0н} = 3$ kgf/cm^2 the efficiency value in regime $K = K_3 = 0.0185$ is greater than zero ($\eta''_{из} = 0.566$); in a case where $p_{г0н} = 4$ kgf/cm^2 in a range of variation in the ejection coefficient from 0.0335 (point f) to $K = K_3 = 0.0483$ efficiency becomes negative. The range of negative values of $\eta''_{из}$ at $p_{г0н} = 4$ kgf/cm^2 corresponds to the physically possible regimes in which total pressure of the subsonic flow of the mixture is less than the total pressure of the ejected gas ($p''_{c03}/p_{г0н} < 1$). In the studied case the flow pattern shown in Fig. 17 develops in the initial part of the mixing chamber.

Analysis of the calculation materials indicate that the supersonic flow of the mixture at the outlet from the mixing chamber at values of $p_{г0н}$ which are equal to 3 and 4 kgf/cm^2 is

not possible at any values of the ejection coefficient, since this would contradict the second law of thermodynamics. Actually, in the range of variation of the ejection coefficient from zero to the value which corresponds to point e (see Fig. 18 and Table 1) the effective work of gas compression is accomplished under a negative energy expenditure, which is absurd (the total pressure of the mixture in these regimes exceeds the total pressure of the ejected liquid, and thus efficiency is less than zero). In the range of variation in the ejection coefficient from values corresponding to points e, to values corresponding to points c (in Fig. 18), the advent of a supersonic flow of the mixture is also impossible, since although $p_{c03} < p_{ж0н}$, the effective work of gas compression exceeds the expended energy ($\eta''_{из} > 1$). Finally, in the range of change of the ejection coefficient from values which correspond to points c (see Fig. 21) to values of $K = K_3$, where the work expended is less than effective work ($\eta'_{из} < 1$), the supersonic flow of the mixture cannot be realized, since it can only develop as a result of an expansion shock, where the entropy of the mixture decreases as it passes through the shock.

Thus, at values of $p_{г0н}$ which exceed the value of the corresponding case, where in regime $W_{г1} = W_{ж1}$ the velocity of the mixture W_{c3} is equal to the speed of sound a_{c3} (in the studied example this value of $p_{г0н}$ is equal to 1.3 kgf/cm^2), the flow of the mixture cannot be supersonic. In this case the cut-off regime of the ejector is the cut-off regime of the mixing chamber.

Now let us examine dependences $\eta_{из}(K)$ for the total-pressure values of the ejected gas $p_{г0н}$ = 1.0; 0.3 and 0.1 kgf/cm^2 (see Fig. 22 and Table 1).

We see that with an increase in the ejection coefficient from zero to K_3 quantity $\eta''_{из}$ increases steadily. The subsonic flow of the mixture, according to the second law of thermodynamics, is possible only in a range of ejection coefficient variation from

zero to values corresponding to points c, at which quantity $\eta''_{из}$ becomes equal to unity. The work of the ejector at greater values of the ejection coefficient in the case of a subsonic flow of the mixture is impossible, since in these regimes $\eta''_{из} > 1$.

A subsonic flow of the mixture when $p_{г0н} = 1.0$, 0.3, and 0.1 kgf/cm^2, as follows from Fig. 22 and Table 1, is theoretically possible only in regimes in which $W_{г1} = W_{ж1}$ ($\eta'_{из} = 1$, points b). When $W_{г1} \neq W_{ж1}$ a supersonic flow is impossible in view of the fact that the efficiency of the ejector exceeds unity, while the entropy of the mixture is less than the sum of entropies of the gas and the liquid at the ejector inlet. Thus, it follows, that for a change in counterpressure from the value corresponding to the zero flow rate of ejected gas (K = 0) to the value which corresponds to the regime in which $W_{г1} = W_{ж1}$, the flow at the outlet from the mixing chamber can be only subsonic [segments of curves $\eta''_{из}(K)$ and $W''_{c3}(K)$, lying to the left of points d in Figs. 19, 20 and 22].

The static pressure of the mixture p''_{c3} in these regimes (unlike the case where $p_{г0н} = 3$ and 4 kgf/cm^2) substantially exceeds the static pressure at the mixing chamber inlet. Thus, for example, when $p_{г0н} = 0.1$ kgf/cm^2 and $\lambda_{г1} = 0.01$ the ratio of static pressures $p_{c3}/p_{г1}$ is equal to 18.4, but when $\lambda_{г1} = 0.1$ (regime $W_{г1} = W_{ж1}$) this ratio is equal to 17.4. This can be explained by the fact that in the studied regimes in the mixing chamber at the boundary of the stream there develops an increasingly extensive supersonic region in the two-phase flow (see Fig. 15), which is transformed into a subsonic flow in the shocks.

When regime $W_{г1} = W_{ж1} = W'_{c3}$ is reached the flow in the outlet part of the mixing chamber becomes completely supersonic (points b in Figs. 19 and 20) and the transition to the subsonic region of the flow is accomplished in the plane shock (points d in the same figures). It is obvious that with a decrease in

counterpressure as compared to counterpressure corresponding to the regime in which velocities $W_{\text{г}1}$ and $W_{\text{ж}1}$ become identical, disturbances no longer penetrate the mixing chamber and the ejection coefficient does not change. The ejector in this case operates in a cut-off regime, which in the studied case is the critical regime.

The distinguishing feature of this critical regime, as already mentioned, is the fact that it develops at subsonic velocities of the gas and liquid streams at the inlet to the mixing chamber as a result of a drastic decrease in the speed of sound in the formation of the two-phase mixture.

Thus, the condition for realizing the studied critical regime when $T_{\text{г0н}} = T_{\text{ж.н}}$ in a case where the liquid does not contain dissolved gases and where saturation pressure is considerably lower than the pressure of the ejected gas, is the equality of the velocities of the gas and liquid streams in the inlet section of the mixing chamber:

$$W_{\text{г}1} = W_{\text{ж}1}. \tag{5.1}$$

Condition (5.1) when $T_{\text{г0н}} = T_{\text{ж.н}}$ gives us the ultimate attainable value for the ejection coefficient:

$$K_{\text{э.пр}} = \alpha \frac{\gamma_{\text{г}1}}{\gamma_{\text{ж}1}} = \frac{\alpha p_{\text{г}1}}{\gamma_{\text{ж}1} R_{\text{г}} T_{\text{ж}}} = \frac{\alpha p_{\text{г0т}} \nu_{\text{г1н}} p(\lambda_{\text{т}1})}{\gamma_{\text{ж}1} R_{\text{г}} T_{\text{ж}}}. \tag{5.2}$$

This condition can be used in qualitative analysis of the characteristics of a vacuum liquid-gas ejector and in estimating its effectiveness. In calculations of the critical regimes of actual ejectors it should be kept in mind that since the process of breaking the liquid streams into drops is incomplete, and due to the effect of friction losses against the walls of the mixing chamber and the peculiarities of the flow in the near-wall regions (particularly separation of the liquid on the walls and the development of reverse current), which are not considered in

the theory, the ratio of velocities $W_{г1}/W_{ж1}$ will be less than unity. For this reason relationship (5.2) can be written as

$$K_з = \psi\alpha \frac{\gamma_{г1}}{\gamma_{ж1}} = \psi \frac{\alpha p_{г1}}{\gamma_{ж1} R_г T_ж} = \psi \frac{\alpha p_{г0н} v_{г1н} p(\lambda_{г1})}{\gamma_{ж1} R_г T_ж}, \qquad (5.3)$$

where quantity

$$\psi = \frac{W_{г1}}{W_{ж1}} < 1 \qquad (5.4)$$

depends on the physical properties and the state parameters of the liquid and the gas, the number and configuration of nozzles, and on the geometrical parameter of the ejector α. This quantity is determined experimentally.

5.3. Vacuum Liquid-Gas Ejector with Supersonic Diffuser

The calculations which have been performed (see Table 1) showed that when the gas which is drawn in has total-pressure values which are less than a certain quantity the actual operational regime of a liquid-gas ejector with a cylindrical mixing chamber and an expanding diffuser is the limiting critical regime. In this regime, which is characteristic of a vacuum liquid-gas ejector, a supersonic flow of the two-phase mixture develops in the outlet section of the mixing chamber. This is transformed into a subsonic flow in the plane shock; in this regime the flow of the mixture for the entire length of the expanding diffuser is supersonic.

The dependences of number M'_{c3} and the pressure recovery coefficient $\nu_{п.с} = p''_{c03}/p'_{c03}$ in the plane shock, located in the outlet section of the mixing chamber, on total pressure of the ejected gas, plotted from the results of calculating the parameters of the mixture for $\alpha = 3.3$; $p_{ж0н} = 5$ kgf/cm^2; $T_{ж.н} = T_{г0н} = 288$°K are shown in Fig. 23.

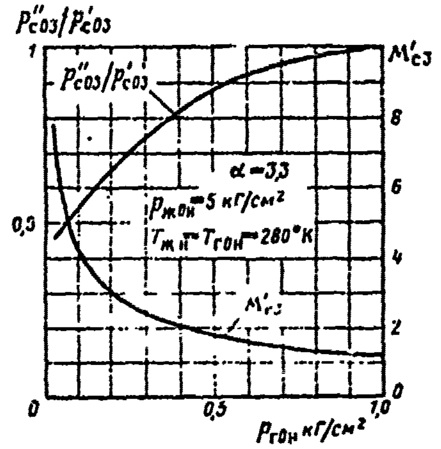


Fig. 23. Number M'_{c3} and coefficient of pressure recovery $\nu_{п.с} = p''_{c03}/p'_{c03}$ in plane shock as a function of total pressure of ejected gas. Designation: кГ/см^2 = kgf/cm^2.

If we examine Fig. 23, we see that with a decrease in the total pressure of the ejected gas number M'_{c3} of the supersonic flow of the mixture in front of the plane shock rises steadily, while the coefficient of pressure recovery in the shock decreases accordingly. In a variation range of $p_{г0н}$ from 1 to 0.5 kgf/cm^2 quantity M'_{c3} rises slightly (from 1.2 to 1.7), and thus losses in the plane shock are relatively low. With a further increase in the pressure of the ejected gas quantity M'_{c3} rises sharply, reaching a value of $M'_{c3} = 7.7$ at $p_{г0н} = 0.03$ kgf/cm^2. The coefficient of pressure recovery in the plane shock decreases in this case to $\nu_{п.с} = 0.46$.

The values of p'_{c03} and $\eta'_{иэ}$ for the limiting critical regime, shown in Table 1, correspond to deceleration of the supersonic flow of the mixture in an ideal supersonic diffuser of the reverse Laval nozzle variety. If we compare these quantities to p''_{c03} and $\eta''_{иэ}$ we find that total-pressure losses in the plane shock at low pressure values of the gas which is drawn in result in a substantial deterioration of the efficiency of the vacuum liquid-gas ejector. Thus, when $p_{г0н} = 0.1$ kgf/cm^2 ($K_3 = 0.39 \times 10^{-3}$) the limiting efficiency of the ejector, when the expanding diffuser is replaced by an ideal supersonic diffuser, rises from 0.34 to 1.0, while total pressure of the mixture increases from 2.0687 to 3.8023 kgf/cm^2. Thus, it follows that one of the basic

means of increasing the efficiency of a vacuum liquid-gas ejector at low pressure values for the gas which is drawn in is to decrease losses which develop during deceleration of the supersonic flow of the two-phase mixture. This can be achieved by replacing the expanding diffuser by a supersonic diffuser which has a throat.

Figure 24 shows the scheme of the vacuum liquid-gas ejector with a supersonic diffuser. When such an ejector is operating in a rated regime the flow of the mixture in the outlet part of the mixing chamber will be supersonic ($M_{c3} = M'_{c3} > 1$). In the convergent part of the diffuser a system of angle shocks develop in which the velocity of the supersonic flow of the two-phase mixture decreases from M'_{c3} to $M'_{c.r} > 1$. The transition into the supersonic region of the flow occurs in the plane shock, located in the throat of the diffuser. In the divergent part of the diffuser there occurs a deceleration of the subsonic flow of the mixture.

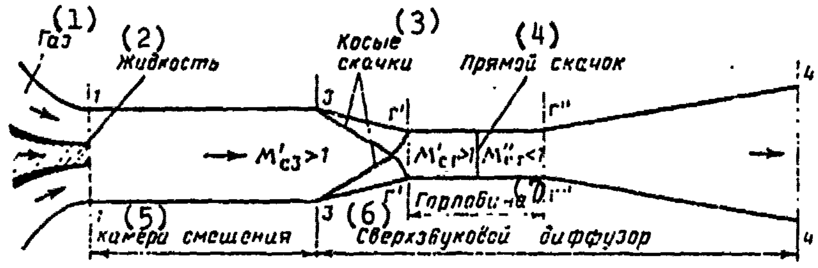


Fig. 24. Scheme of vacuum liquid-gas ejector with supersonic diffuser.
KEY: (1) Gas; (2) Liquid; (3) Angle shocks; (4) Plane shock; (5) Mixing chamber; (6) Supersonic diffuser; (7) Throat.

The total pressure of the subsonic flow of the mixture $p''_{c04(св.л)}$ at the outlet from a supersonic diffuser operating in a rated regime can be represented in the form of

$$p''_{c04(св.л)} = p'_{c03}\nu_{r',3}\nu_{r'',r'}\nu_{4,r''}, \qquad (5.5)$$

where

$$\nu_{\Gamma',3} = p'_{c0\Gamma'}/p'_{c13};\ \nu_{\Gamma'',\Gamma'} = p''_{c0\Gamma''}/p'_{c0\Gamma'} = \nu(\lambda'_{c.\Gamma'});\ \nu_{4,\Gamma''} = p''_{c04}/p''_{c0\Gamma''}$$

are the coefficients of pressure recovery in the convergent part of the supersonic diffuser, in the plane shock developing in the throat, and in the divergent part of the diffuser for a subsonic flow, respectively.

Analysis of (5.5) shows that at assigned values for the relative area of the cross section of the throat $\bar{f}_{\Gamma} = f_{\Gamma}/F$, as well as the parameters of state and reduced velocity of the supersonic flow of the mixture at the outlet from the mixing chamber, product $\nu_{\Gamma',3}\nu(\lambda'_{c.\Gamma'})$ in the variation range of value $\nu_{\Gamma',3}$, which is of practical interest to us, is only very slightly dependent on this value. This can be explained by the fact that with a decrease in value $\nu_{\Gamma',3}$ the velocity of the supersonic flow of the mixture in the throat of the diffuser and, consequently, losses in the plane shock.

Thus, to simplify calculations we can assume, with a degree of accuracy sufficient for practical purposes, that the deceleration of the supersonic flow in the convergent part of the diffuser occurs without total-pressure losses. Relationship (5.5) is in this case written as

$$p''_{c04\,(\text{сж.д})} = p'_{c03}\nu(\lambda'_{c.\Gamma'})\nu_{4,\Gamma''}. \qquad (5.6)$$

The total pressure of the mixture at the outlet from the ejector in the case of an expanding (subsonic) diffuser can, according to the discussion above, be represented in the form of

$$p''_{c04\,(\text{дозв.д})} = p'_{c03}\nu(\lambda'_{c3})\nu_{4,3}, \qquad (5.7)$$

where $\nu(\lambda'_{c3})$ is the coefficient of pressure recovery in the plane shock, located in the outlet section of the mixing chamber.

In expressions (5.6) and (5.7) we : ' ,

$$\frac{p_{c0(\text{св.х})}}{p_{c0(\text{косз.х})}}=\frac{\nu\left(\lambda'_{c.\Gamma'}\right)\nu_{4.\Gamma'}}{\nu\left(\lambda'_{c3}\right)\nu_{4,3}}\approx\frac{\nu\left(\lambda'_{c.\Gamma'}\right)}{\nu\left(\lambda'_{c3}\right)}. \tag{5.8}$$

From this it follows that when a vacuum liquid-gas ejector is working in the most advantageous limiting critical regime, replacement of the expanding subsonic diffuser by a supersonic diffuser with a throat will result in an increase in total pressure of the subsonic flow of the mixture, other conditions being equal, by a number of times which is approximately equal to the ratio of pressure recovery coefficient in the direct shocks, located in the throat of the supersonic nozzle and in the outlet section of the mixing chamber, respectively.

Obviously the lower the relative area of the cross section in the throat of the supersonic diffuser, the greater will be the advantage gained by using it. In the case of an unregulated supersonic diffuser, which can be brought to the rated regime by simply changing counterpressure, the minimal value of the relative area of the throat $\bar{f}_{\Gamma\ min}$ can be found from the condition that in the presence of a plane shock in the outlet section of the mixing chamber the velocity of the mixture in the throat is equal to the critical speed of sound. Quantity $\bar{f}_{\Gamma\ min}$ is calculated as follows:

1) using the ejection equation system for the limiting critical regime we find quantities p''_{c03}, T''_{c03}, K, R_c, $\bar{G}_c = G_c/F$, and p'_{c03}, W'_{c3}, γ'_{c3};

2) if we consider that the flow of the mixture in the case of a vacuum liquid-gas ejector can be regarded as approximately isothermic and, if we assume that there are no losses in the convergent part of the diffuser in the subsonic flow, then from equation (1.51) we determine the critical speed of sound $a_{c.\kappa} = W_{c.\Gamma}$ from the quantities p''_{c03}, T''_{c03}, K, R_c which have been found;

3) from expressions (1.45) and (1.43) we calculate quantities $p_{c.\kappa} = p_{c.\Gamma}$, $\gamma_{c.\kappa} = \gamma_{c.\Gamma}$;

4) from formula

$$\bar{f}_{\Gamma\,min} = \bar{G}_c / \gamma_{c.\kappa} a_{c.\kappa} \tag{5.9}$$

we find the sought value of the minimal relative area of the throat of the unregulated supersonic diffuser.

After determining $\bar{f}_{\Gamma\ min}$ we can find the total pressure of the mixture at the outlet from an ejector working in the limiting critical regime, when the plane shock is located in the throat of the diffuser.

1. If we assume that $p'_{c0\Gamma} = p'_{c03}$ (see above), then from equation

$$\frac{1}{2\gamma}\left\{\frac{\bar{G}_c}{p'_{c.\Gamma}\bar{f}_{\Gamma\,min}}\left[b+\frac{p'_{c.\Gamma}}{\gamma_{\text{ж}}(K+1)}\right]\right\}+b\ln p'_{c.\Gamma}+\frac{p'_{c.\Gamma}}{\gamma_{\text{ж}}(K+1)}=$$
$$=b\ln p'_{c0\Gamma}+\frac{p'_{c0\Gamma}}{\gamma_{\text{ж}}(K+1)} \tag{5.10}$$

we determine the static pressure $p'_{c.\Gamma}$ of the supersonic flow of the mixture in the throat of the diffuser, and then from equations (1.43) and (1.45) - quantities $\gamma'_{c.\Gamma}$, $a'_{c.\Gamma}$.

2. According to formula $W'_{c.\Gamma} = \bar{G}_c/\gamma'_{c.\Gamma}\bar{f}_{\Gamma\ min}$ we calculate the velocity of the supersonic flow of the mixture in the throat and number $M'_{c.\Gamma} = W'_{c.\Gamma}/a'_{c.\Gamma}$.

3. Using relationship (1.65), (1.66), (1.67) and (1.68) we find quantities $M''_{c.\Gamma}$, $p''_{c.\Gamma}$, $\gamma''_{c.\Gamma}$, $a''_{c.\Gamma}$, and $W''_{c.\Gamma}$, which correspond to the subsonic flow of the mixture in the throat behind the plane shock.

4. From equation (1.50) we determine the total pressure of the subsonic flow of the mixture in the outlet section of the throat and the total pressure of the mixture at the outlet from the ejector $p_{c04(\text{св.д})} = p''_{c0\Gamma''}\nu_{4,\Gamma''}$.

Total pressure of a mixture at the outlet from a vacuum-liquid gas ejector with an unregulated supersonic diffuser when $\bar{f}_{\Gamma} = \bar{f}_{\Gamma\ min}$, as indicated by the calculation and the experiment (see [8]), substantially (1.5-2 times) exceeds the total pressure of the mixture at the outlet from the same ejector with an expanding diffuser.

5.4. Estimating Value of Total Pressure of Mixture in the Case of a Vaccum Liquid-Gas Ejector with Expanding Diffuser

The calculations indicated that at assigned values of α, $\bar{\delta}$, $p_{\text{ж}0\text{н}}$, $T_{\text{ж.н}}$ and $T_{\Gamma 0\text{н}}$ the total pressure of the subsonic flow of the mixture p''_{c03} in the outlet section of the mixing chamber of a vacuum liquid-gas ejector working in a limiting critical regime changes only slightly with a change in the ejection coefficient and, consequently, total pressure of the ejected gas. For this reason it is possible to estimate the total pressure of a mixture in a vacuum liquid-gas ejector according to the Borda-Carnot formula for a flow of an incompressible liquid in a region of sudden tube expansion.

If we assume that K = 0, $p_1 = p_{\text{ж}1} = p_{s\ \text{н}}$, and consider expression (2.3), then we get

$$p^{*}_{c03}=\nu_{\text{ж}1\text{н}}p_{\text{ж}0\text{н}}\left[1-\left(1-\frac{p_{s\,\text{н}}}{\nu_{\text{ж}1\text{н}}\,p_{\text{ж}0\text{н}}}\right)\left(\frac{\alpha+\bar{\delta}}{\alpha+1}\right)^{2}\right]. \qquad (5.11)$$

The total pressure of the mixture at the outlet from the ejector can be found from formula

$$p^{*}_{c04}=\nu_{4,3}\nu_{\text{ж}1\text{н}}p_{\text{ж}0\text{н}}\left[1-\left(1-\frac{p_{s\,\text{н}}}{\nu_{\text{ж}1\text{н}}p_{\text{ж}0\text{н}}}\right)\left(\frac{\alpha+\bar{\delta}}{\alpha+1}\right)^{2}\right]. \qquad (5.12)$$

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